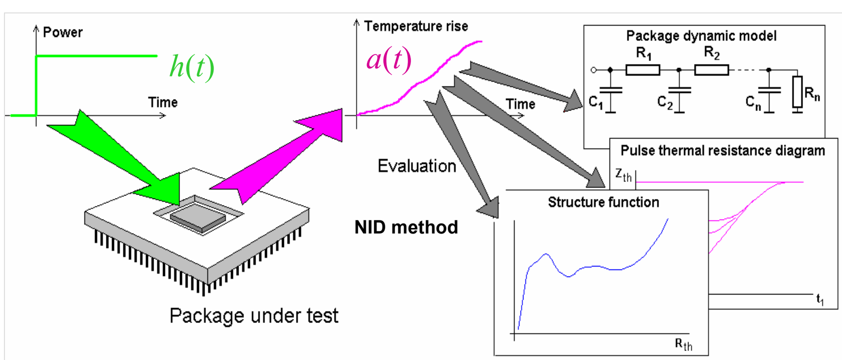
Gallop in the linear theory

- Discussed many times before
- Skip it for now, but I will upload to the server for reference

Thermal transient testing



The **measured** $a(t)$ **response function** is **characteristic** to the package.

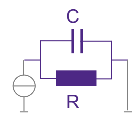
The features of the *chip* + *package* + *environment* structure can be extracted from it.

Step-response functions

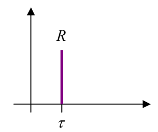
The form of the step-response function

- for a ***single*** RC stage:

$$a(t) = R \cdot [1 - \exp(-t/\tau)]$$



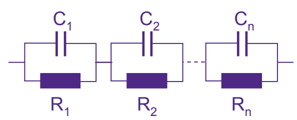
$$\tau = R \cdot C$$



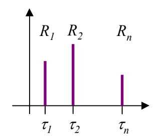
characteristic values: R magnitude and τ time-constant

- for a ***chain*** of n RC stages:

$$a(t) = \sum_{i=1}^{n} R_i \cdot [1 - \exp(-t/\tau_i)]$$



$$\tau_i = R_i \cdot C_i$$



characteristic values: set of R_i magnitudes and τ_i time-constants

If we know the R_i and τ_i values, we know the system.

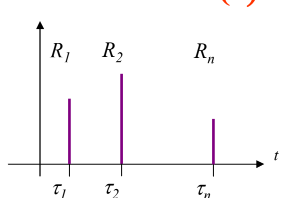
Step-response functions

- for a ***distributed RC system***:

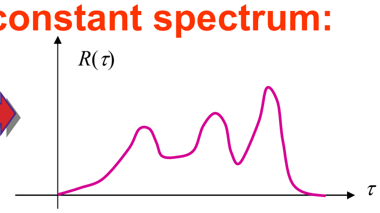
$$n \Rightarrow \infty \qquad \sum_{i=1}^{n} \Rightarrow \int_0^{\infty}$$

$$a(t) = \sum_{i=1}^{n} R_i \cdot [1 - \exp(-t/\tau_i)] \Rightarrow a(t) = \int_0^{\infty} R(\tau)[1 - \exp(-t/\tau)]d\tau$$

characteristic: **$R(\tau)$ time-constant spectrum:**



discrete set of R_i and τ_i values



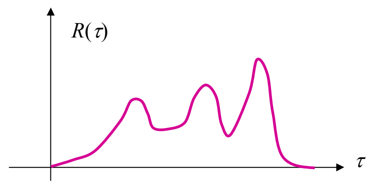
continuous $R(\tau)$ spectrum

If we know the $R(\tau)$ function, we know the distributed RC system.

Time-constant spectrum

Discrete RC stages ⇒ *discrete set of R_i and τ_i values*
Distributed RC system ⇒ *continuous $R(\tau)$ function*

$$a(t) = \int_0^\infty R(\tau)[1 - \exp(-t/\tau)]d\tau$$



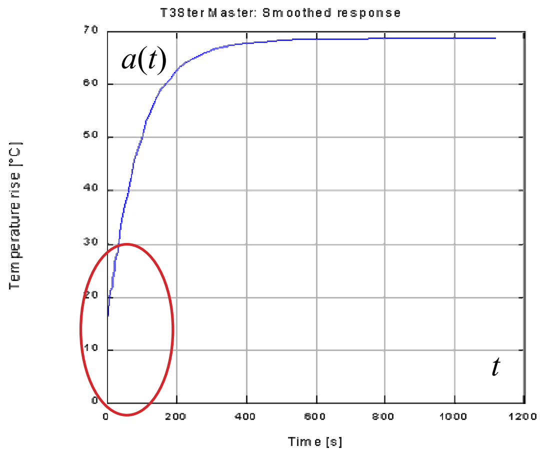
If we know the $R(\tau)$ function, we know the system.

$R(\tau)$ is called the **time-constant spectrum**.

Practical problem

- The range of **possible time-constant values** in thermal systems spans over **5..6 decades** of time
  - 100μs ..10ms range: semiconductor chip / die attach
  - 10ms ..50ms range: package structures beneath the chip
  - 50ms ..1 s range: further structures of the package
  - 1s ..10s range: package body
  - 10s ..10000s range: cooling assemblies
- Wide time-constant range ⇒ data acquisition problem during measurement/simulation: ***what is the optimal sampling rate?***

Practical problem (cont.)

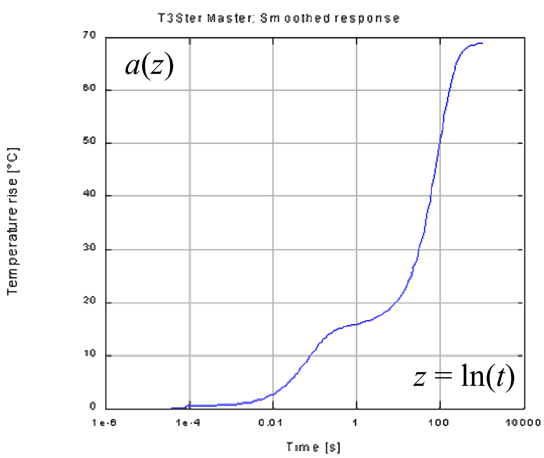


Measured unit-step response of an MCM shown in linear time-scale

Nothing can be seen below the 10s range

Solution: equidistant sampling on *logarithmic time scale*

Using logarithmic time-scale



Measured unit-step response of an MCM shown in linear time-scale

Details in all time-constant ranges are seen

Instead of t time we use $z = \ln(t)$ **logarithmic time**

Step-response in log time

- Switch to logarithmic time scale:

$$a(t) \Rightarrow a(z) \textit{ where } z = \ln(t)$$

$a(z)$ is called*
- **heating curve** or
- **thermal impedance curve**

- **Zth curve**

*Sometimes $P \cdot a(z)$ is called heating curve in the literature.

Step-response in log time

- **Zth curve**
- **Use the chain rule !** $\frac{dy}{dz} = \frac{dy}{dt} \cdot \frac{dt}{dz}$

- Using the $z = \ln(t)$ transformation it can be proven that

$$\frac{d}{dz} a(z) = \int_0^\infty R(\zeta)\left[\exp(z - \zeta - \exp(z - \zeta))\right] d\zeta$$

*Sometimes $P \cdot a(z)$ is called heating curve in the literature.

Step-response in log time

- Note, that $da(z)/dz$ is in a form of a convolution integral: $$\frac{d}{dz}a(z)=\int_0^\infty R(\zeta)\left[\exp(z-\zeta-\exp(z-\zeta))\right]d\zeta$$

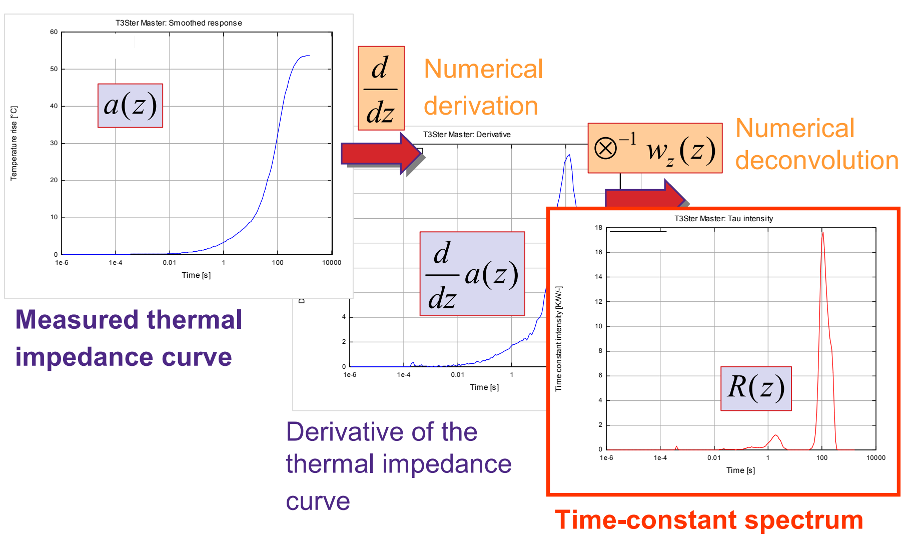
  Introducing the $w_z(z)=\exp(z-\exp(z))$ function:

$$\frac{d}{dz}a(z)=\int_0^\infty R(\zeta)\cdot w_z(z-\zeta)d\zeta$$

$$\frac{d}{dz}a(z)=R(z)\otimes w_z(z)$$

- From $a(z)$ $R(z)$ is obtained as: $$R(z)=\left[\frac{d}{dz}a(z)\right]\otimes^{-1}w_z(z)$$

Extract the time-constant spectrum



Extract the time-constant spectrum

$a(z)$ Must be noise free, must have high time resolution (e.g. 200 points/decade)

$\frac{d}{dz}$ Numerical derivation should be accurate: high order techniques yield better results.

Danger of noise enhancement ⇒ filtering ⇒ loss of ultimate resolution in the time-constant spectrum

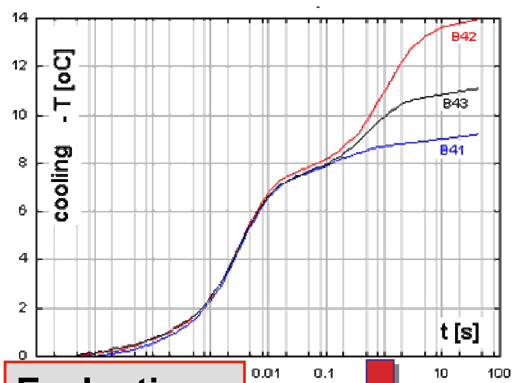
$\otimes^{-1} w_z(z)$ Numerical deconvolution: Bayes-iteration (for driving point impedance only), frequency-domain inverse filtering (both for driving point and transfer impedances)

$R(z)$ False values with small magnitude can be present due to noise enhancement in the procedure. ***Negative values represent a transfer impedance.***

Using time-constant spectra

- The time-constant spectrum gives hint for the time-domain behavior of the system for experts
- Time-constant spectra can be further processed and turned into ***other characteristic functions***
- These functions are called ***structure functions***

Models of thermal impedances

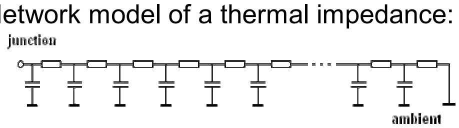


heating or cooling curves

Normalized to 1W dissipation: **thermal impedance curve**

Evaluation:

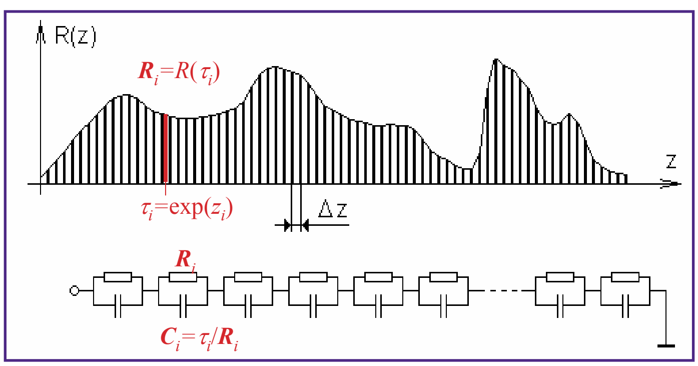
Network model of a thermal impedance:



Interpretation of the impedance model: **STRUCTURE FUNCTIONS**

Structure functions 1

- Discretization of $R(z)$ ⇒ RC network model in **Foster canonic form** (instead of ∞ spectrum lines, 100..200 RC stages)



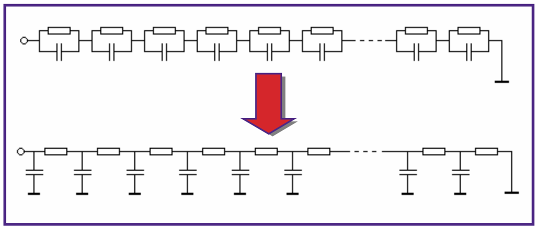
- A discrete RC network model is extracted ⇒ name of the method: **NID - network identification by deconvolution**

Structure functions 2

- The Foster model network is just a theoretical one, does not correspond to the physical structure of the thermal system:

  thermal capacitance exists towards the ambient (thermal "ground") only

- The model network has to be converted into the **Cauer canonic form**:

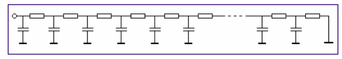


- Ronald Martin Foster
- 1896 - 1998

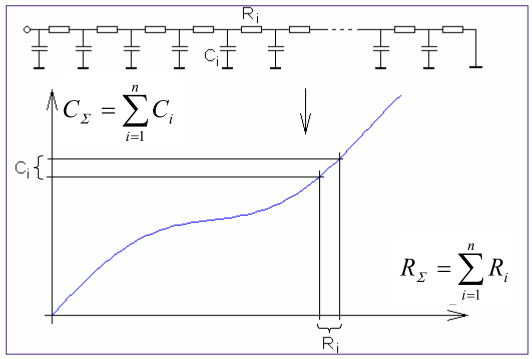
- Wilhelm Cauer
- 1900 - 1945

Structure functions 3

- The identified RC model network in the Cauer canonic form now **corresponds to the physical structure**, but

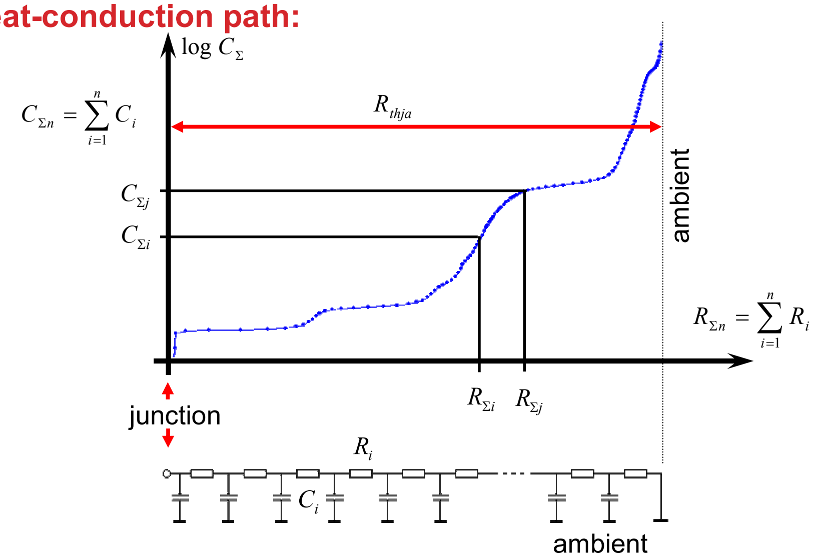


- it is very hard to interpret its "meaning"
- Its graphical representation helps:
- This is called ***cumulative structure function***



Structure functions 4

The *cumulative structure function* is the *map* of the heat-conduction path:

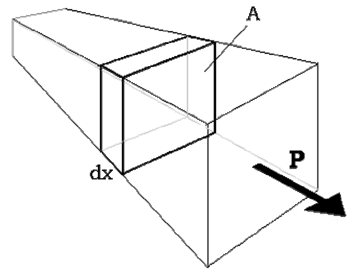


Differential structure functions

- The *differential structure function* is defined as the derivative of the cumulative thermal capacitance with respect to the cumulative thermal resistance

$$K(R_\Sigma) = \frac{dC_\Sigma}{dR_\Sigma}$$

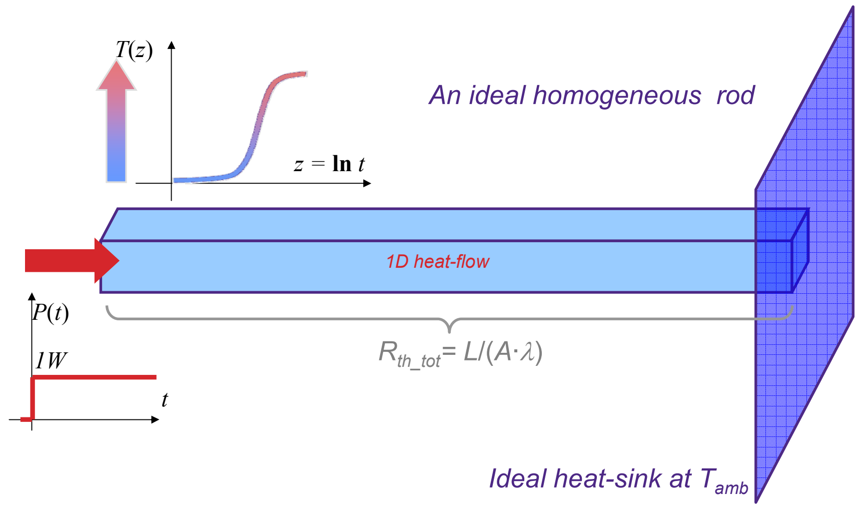
$$K(R_\Sigma) = \frac{cAdx}{dx/\lambda A} = c\lambda A^2$$



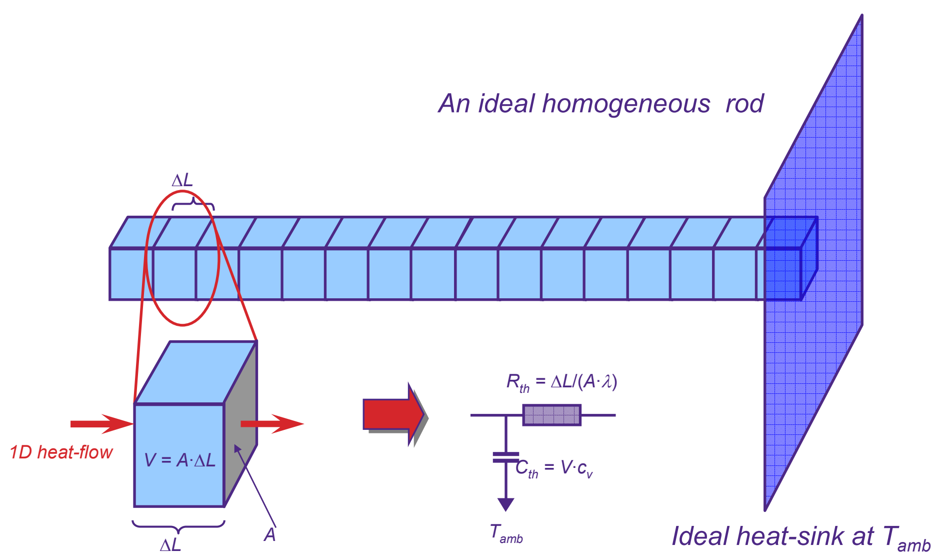
- K is proportional to the square of the cross sectional area of the heat flow path.

What do structure functions tell us and how?

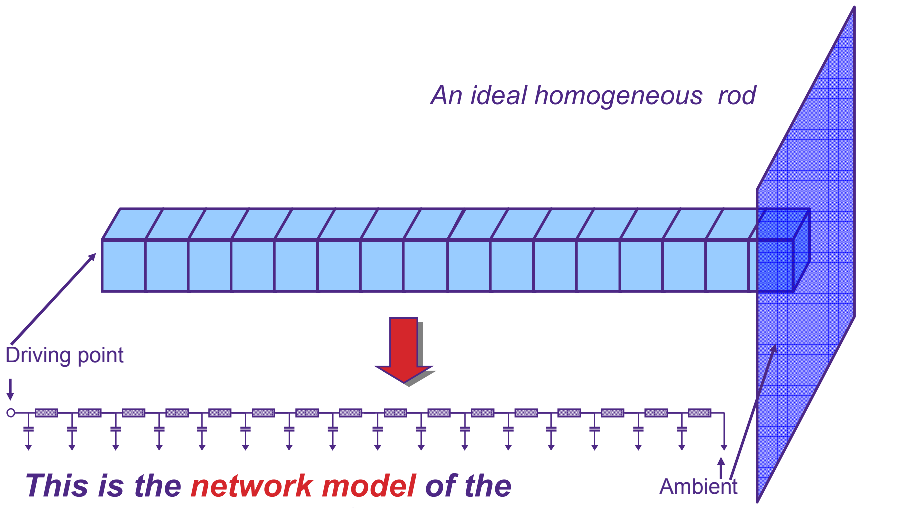
A hypothetic example 1



A hypothetic example 2



A hypothetic example 3

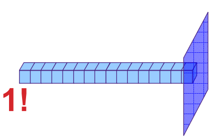


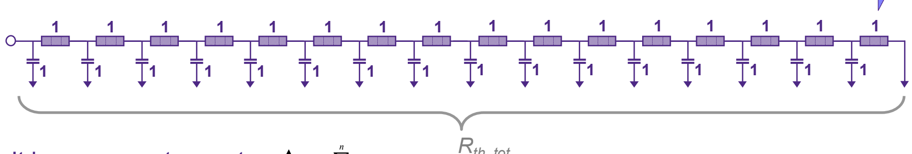
This is the network model of the thermal impedance of the rod

Ideal heat-sink at T_{amb}

A hypothetic example 4

Let us assume ΔL, *A* and *material parameters* such, that **all element values in the model are 1!**

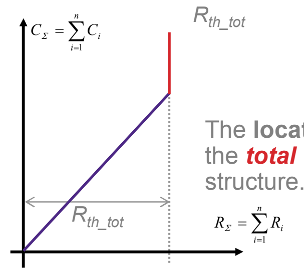




It is very easy to create the **cumulative structure function:**

y=x - a straight line

There must be a singularity when we reach the ideal heat-sink.

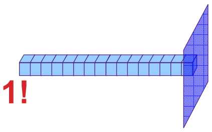


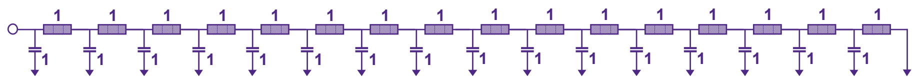
The **location of the singularity** gives the ***total thermal resistance*** of the structure.

$R_\Sigma = \sum_{i=1}^{n} R_i$

A hypothetic example 5

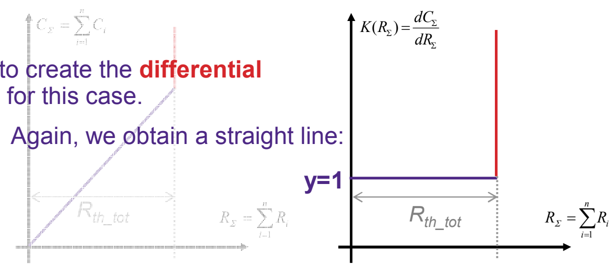
Let us assume ΔL, *A* and *material parameters* such, that **all element values in the model are 1!**





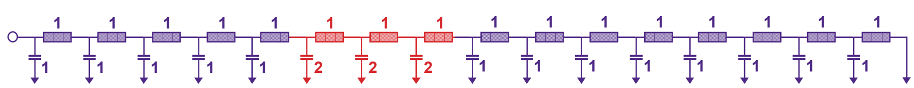
It is also very easy to create the **differential structure function** for this case.

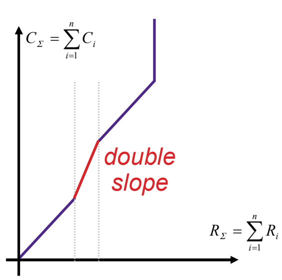
Again, we obtain a straight line:



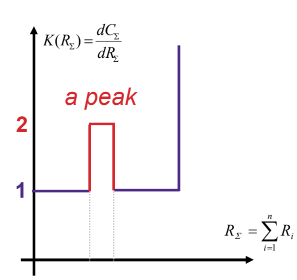
A hypothetic example 6

What happens, if e.g. in a certain section of the structure model all capacitance values are equal to 2?





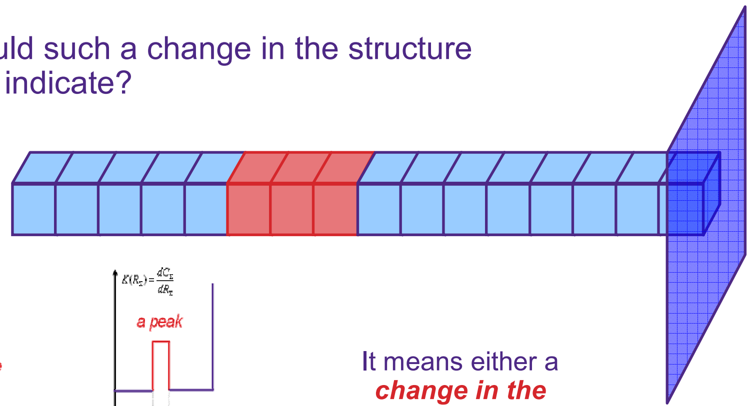
Cumulative structure function

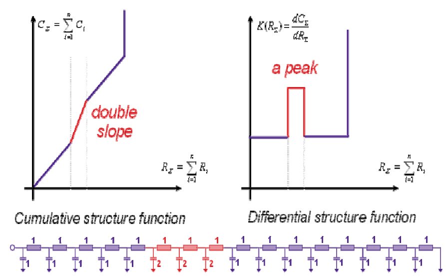


Differential structure function

A hypothetic example 7

What would such a change in the structure functions indicate?

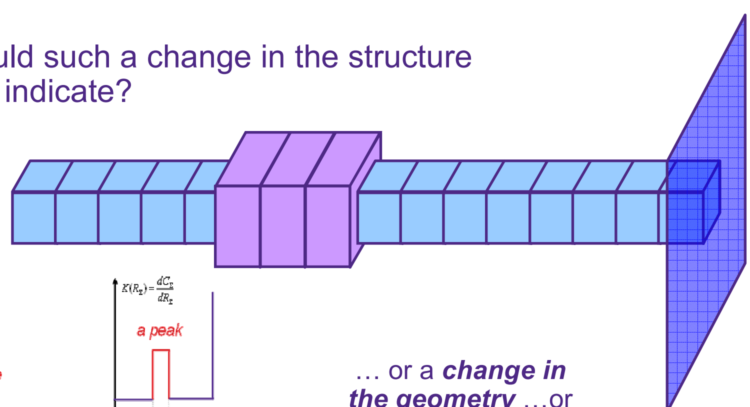


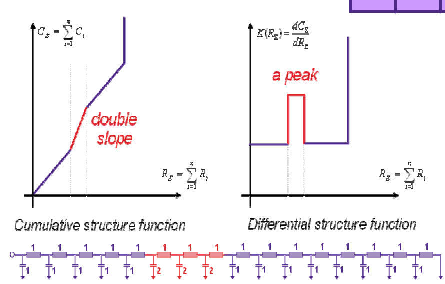


It means either a ***change in the material properties…***

A hypothetic example 8

What would such a change in the structure functions indicate?

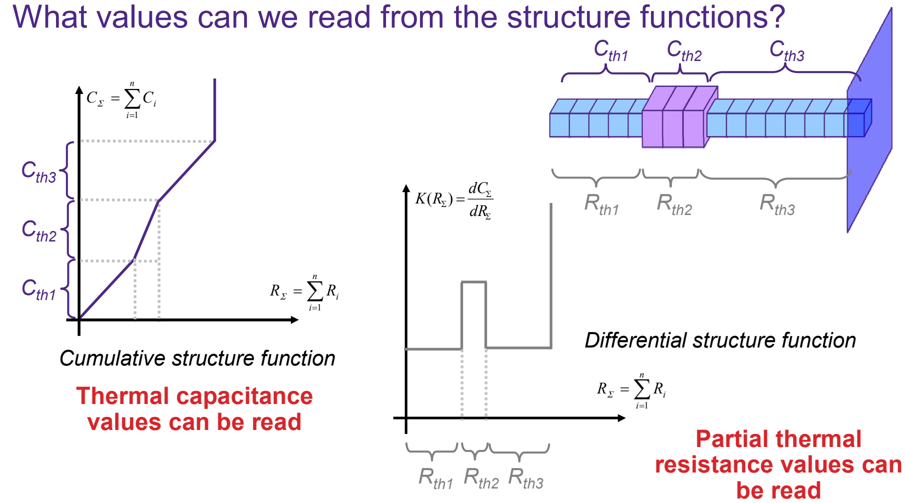




… or a ***change in the geometry*** …or ***both***

A hypothetic example 9

What values can we read from the structure functions?

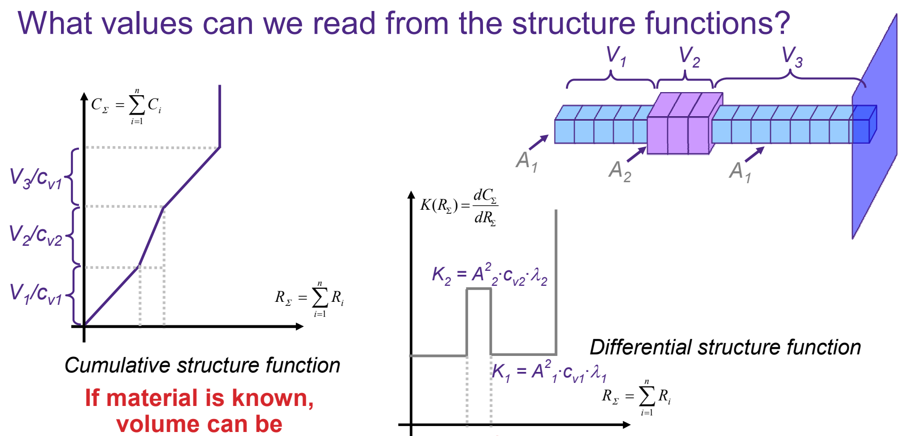


Thermal capacitance values can be read

Partial thermal resistance values can be read

A hypothetic example 10

What values can we read from the structure functions?

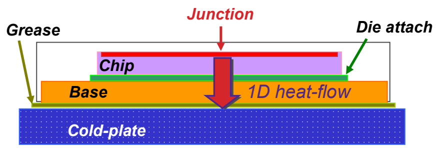


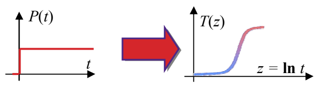
If material is known, volume can be identified.
If volume is known, volumetric thermal capacitance can be identified.

If material is known, cross-sectional area can be identified.
If cross-sectional area is known, material parameters ($c_V \cdot \lambda$) can be identified.

Model of 1D flow: *structure func.*

- *Structure functions*: thermal capacitance vs. thermal resistance maps of the heat-flow path

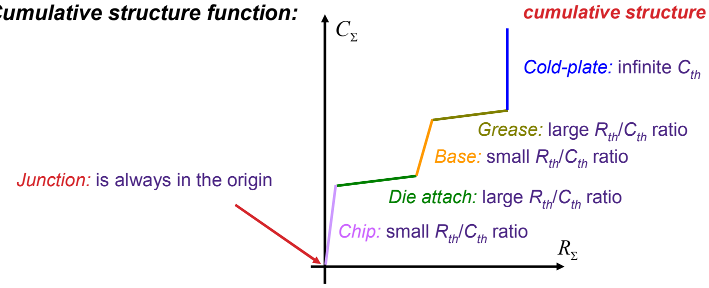




*We measure the **thermal transient at the junction**...*

*...and we convert it into the **cumulative structure function:***

Cumulative structure function:

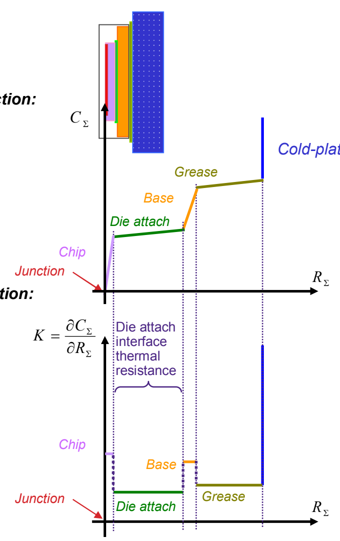


Model of 1D flow: *structure func.*

- *Structure functions*: maps of the heat-flow path

Cumulative structure function:

Differential structure function:



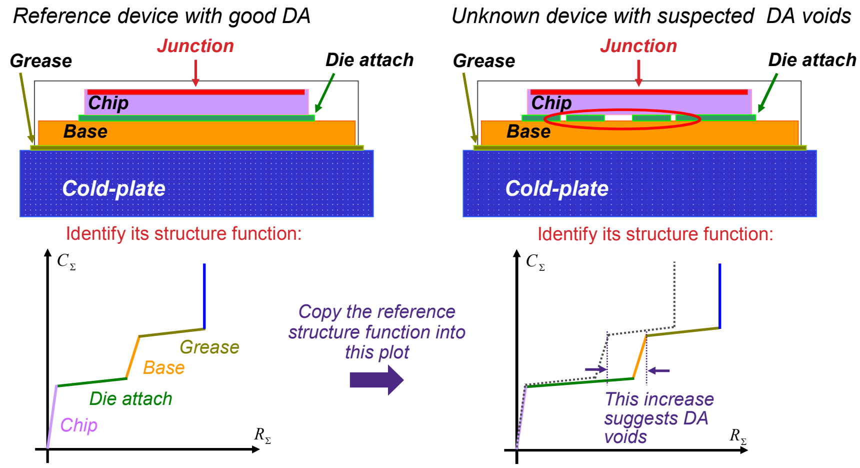
The heat-flow path can be well characterized e.g. by partial thermal resistance values

The R_{thDA} value is derived entirely from the junction temperature transient.

No thermocouples are needed.

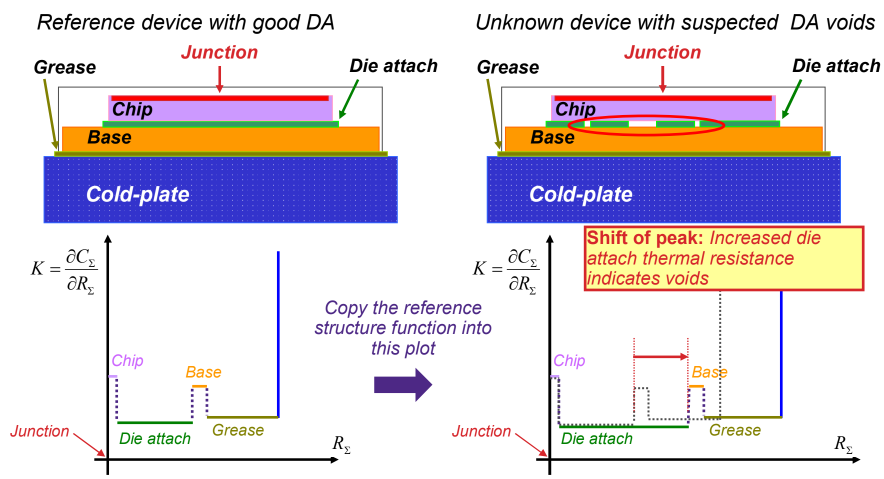
DA testing with structure functions

- *Structure functions*: tools for *structural analysis* and *comparison*



DA testing with structure functions

- *Structure functions*: tools for *structural analysis* and *comparison*

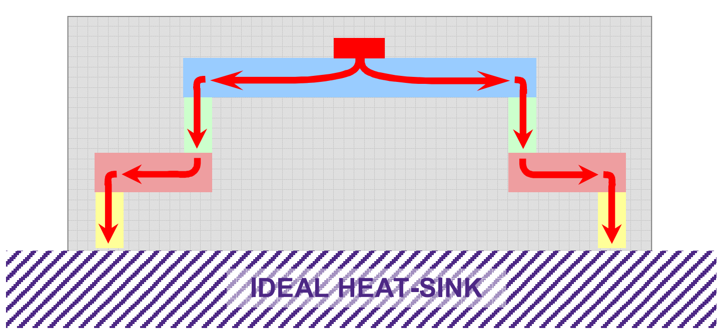


Some conclusions

- Structure functions are direct models of one-dimensional heat-flow
  - longitudinal flow (like in case of a rod)
- Also, structure functions are direct models of "essentially" 1D heat-flow, such as
  - radial spreading in a disc (1D flow in polar coordinate system)
  - spherical spreading
  - conical spreading
  - etc.
- Structure functions are "reverse engineering tools": geometry/material parameters can be identified with them

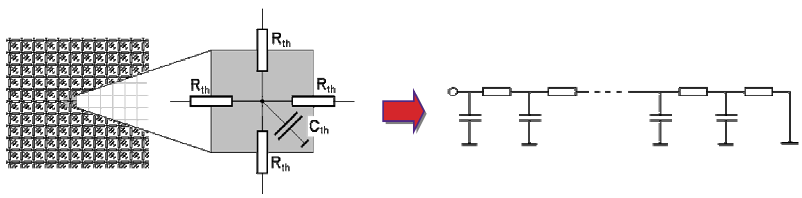
Some conclusions (cont.)

- In many cases a complex heat-flow path can be partitioned into essentially 1D heat-flow path sections connected in series:



Some conclusions (cont.)

- In case of complex, 3D streaming the derived model has to be considered as an *equivalent physical structure providing the same thermal impedance as the original structure.*



Some special properties

- For "ideal" cases structure functions can be given even by analytical formulae
  - for a rod:

$$C_\Sigma = const \cdot R_\Sigma$$

  - for radial spreading in a disc of w thickness and λ thermal conductivity:

$$\lambda w = \frac{1}{4\pi}\frac{\ln(C_{th2}/C_{th1})}{R_{th2}-R_{th1}}$$