Simple Theory of the Ballistic Nanotransistor

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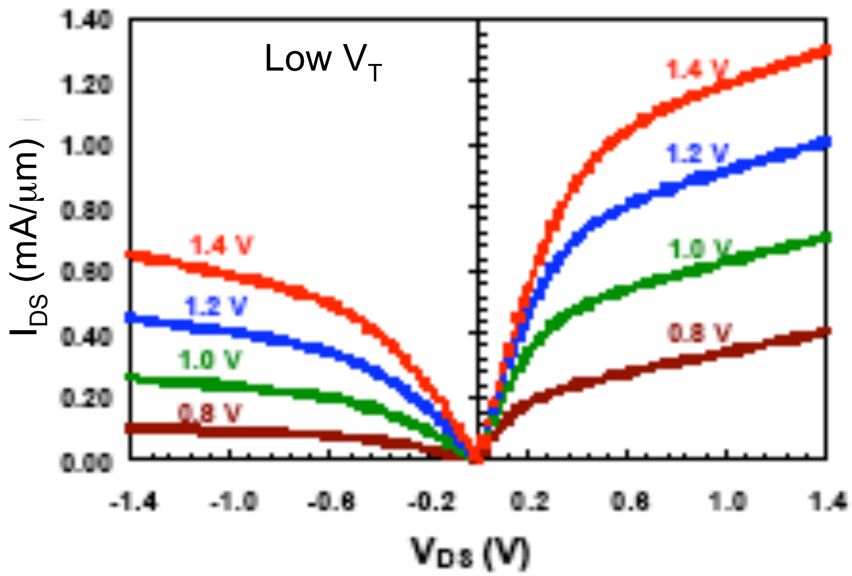
outline

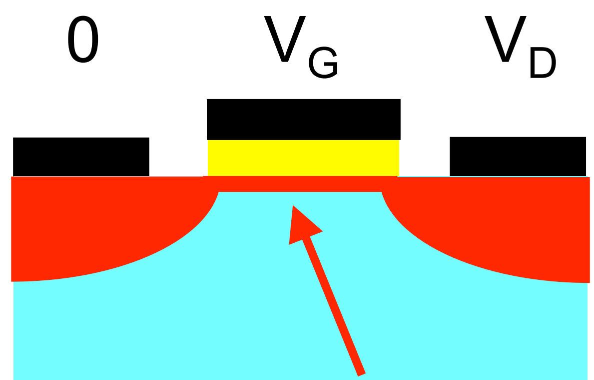
I) Traditional MOS theory
II) A "bottom-up" approach
III) The ballistic nanotransistor
IV) Discussion
V) Summary

130 nm technology (L_G = 60 nm)



Intel Technical J., Vol. 6, May 16, 2002.

MOSFET IV: low V_{DS}

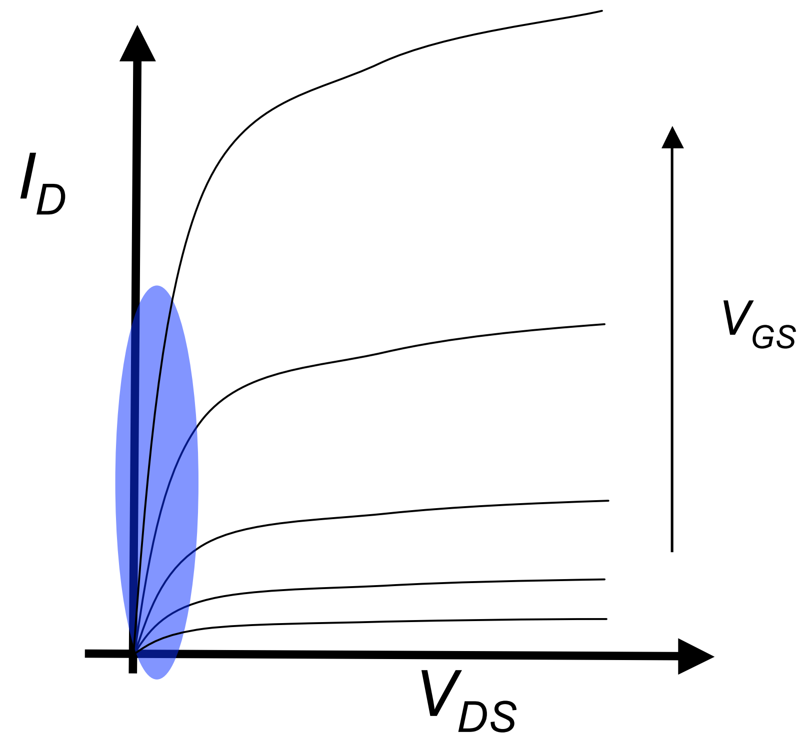


$$Q_i(x) = -C_{ox}\left(V_{GS} - V_T - V(x)\right)$$

$$I_D = W\,Q_i(x)\upsilon_x(x) = W\,Q_i(0)\upsilon_x(0)$$

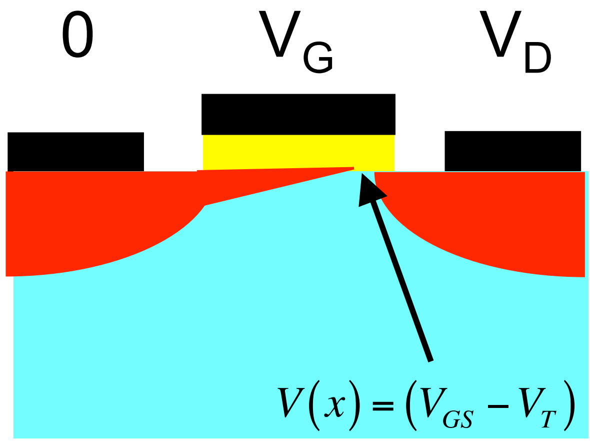
$$I_D = W\,C_{ox}\left(V_{GS} - V_T\right)\mu_{eff}\mathcal{E}_x$$

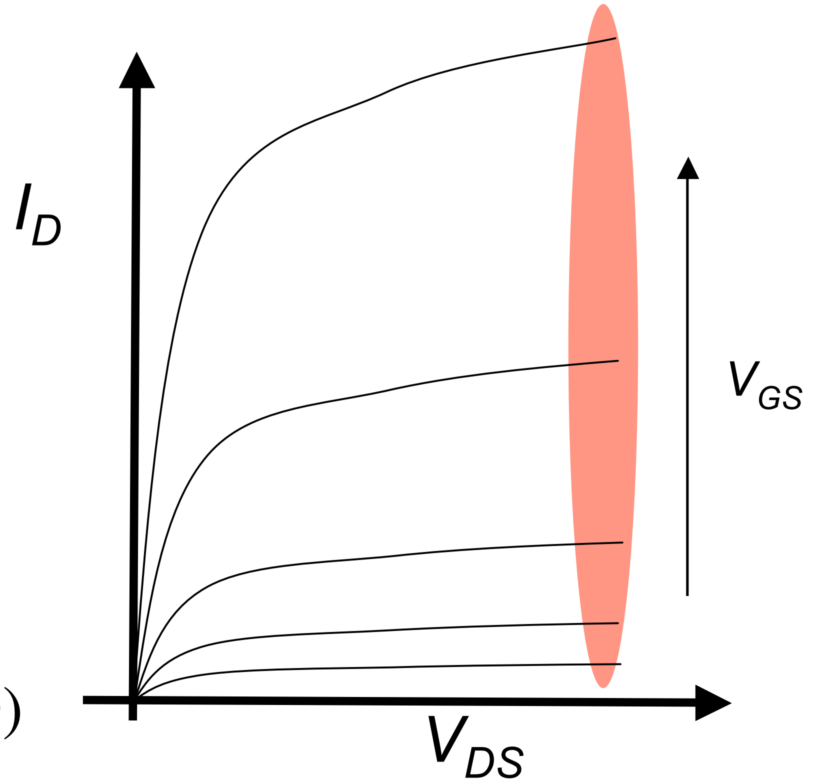
$$\mathcal{E}_x = \frac{V_{DS}}{L}$$



$$I_D = \frac{W}{L}\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

MOSFET IV: high V_{DS}





$$I_D = W\, Q_i(x)\upsilon_x(x) = W\, Q_i(0)\upsilon_x(0)$$

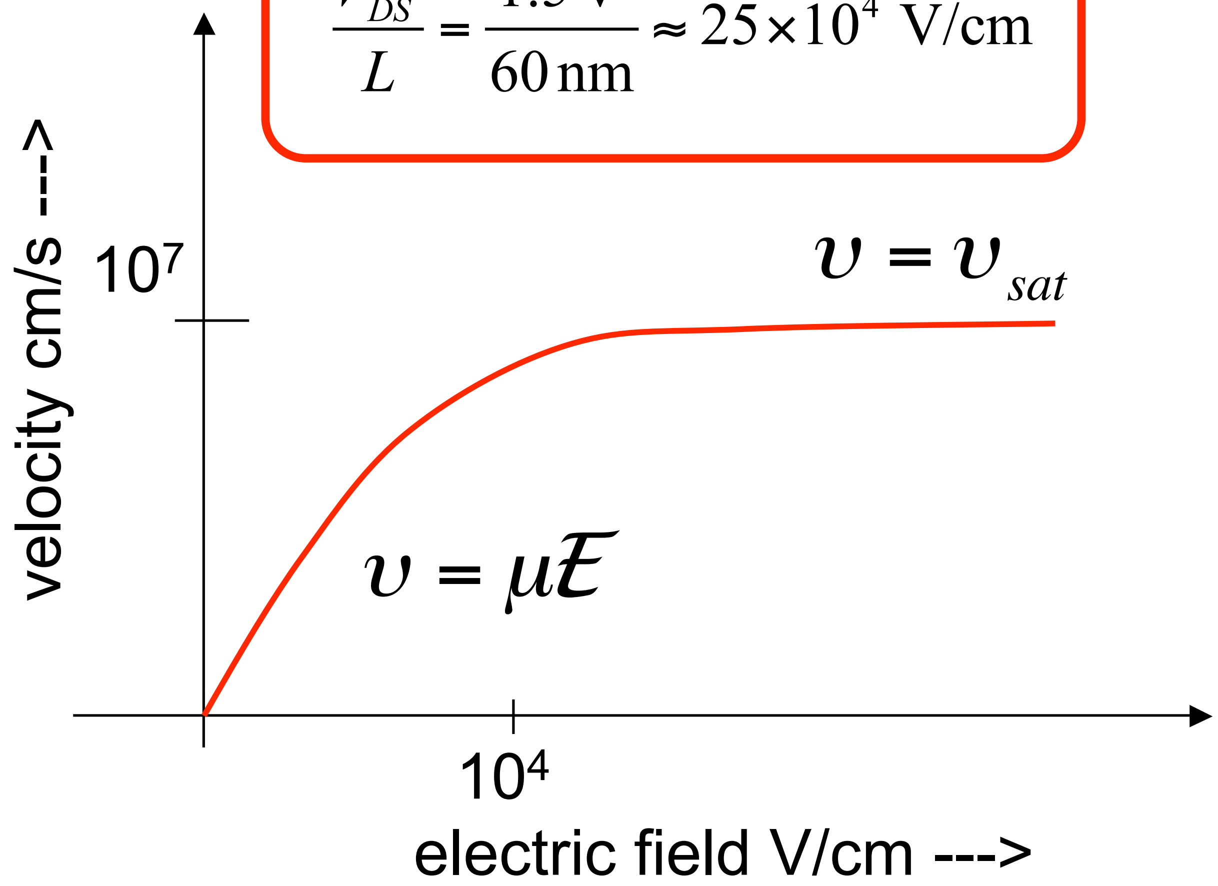
$$I_D = W\, C_{ox}(V_{GS} - V_T)\mu_{eff}\mathcal{E}_x$$

$$\mathcal{E}_x \approx \frac{V_{GS} - V_T}{L}$$

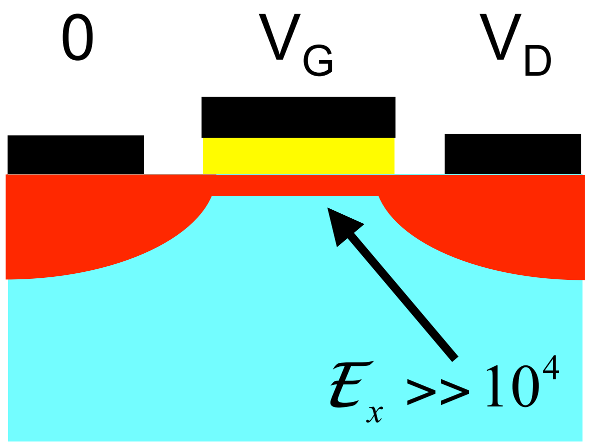
$$I_D = \frac{W}{2L}\mu_{eff} C_{ox}(V_{GS} - V_T)^2$$

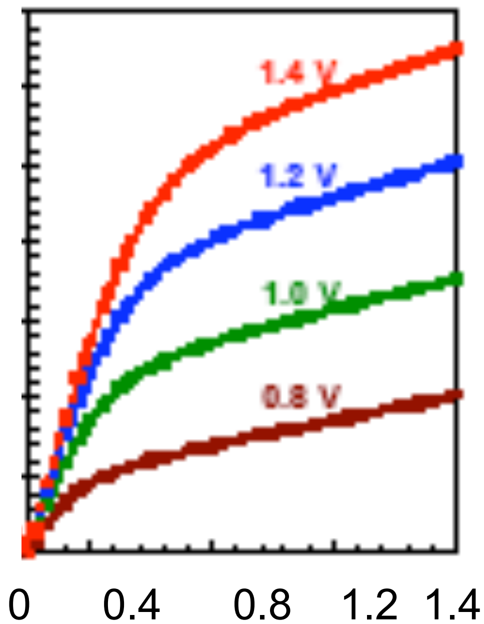
velocity saturation

$$\frac{V_{DS}}{L} = \frac{1.5\,\text{V}}{60\,\text{nm}} \approx 25\times10^{4}\ \text{V/cm}$$



MOSFET IV: velocity saturation



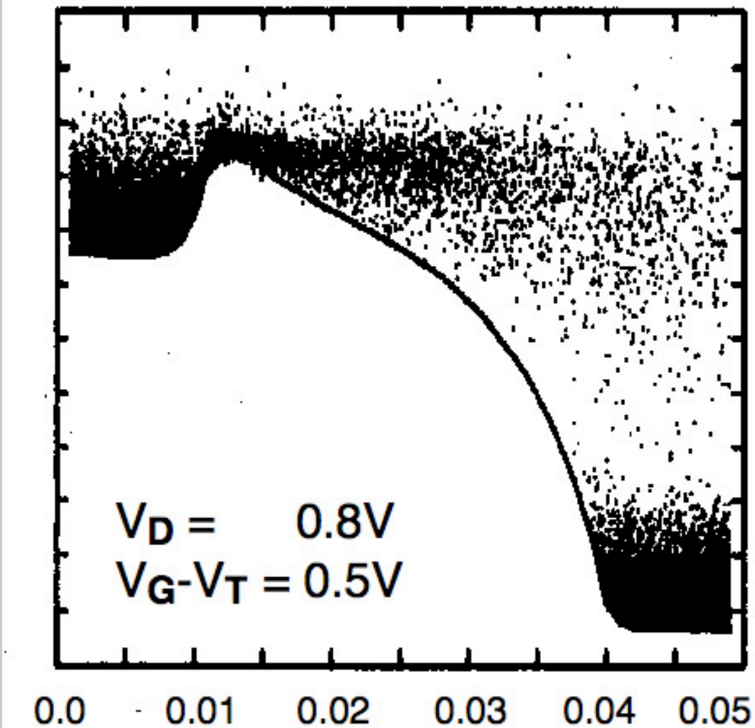


$$I_D = W\,Q_i(x)\upsilon_x(x) = W\,Q_i(0)\upsilon_x(0)$$

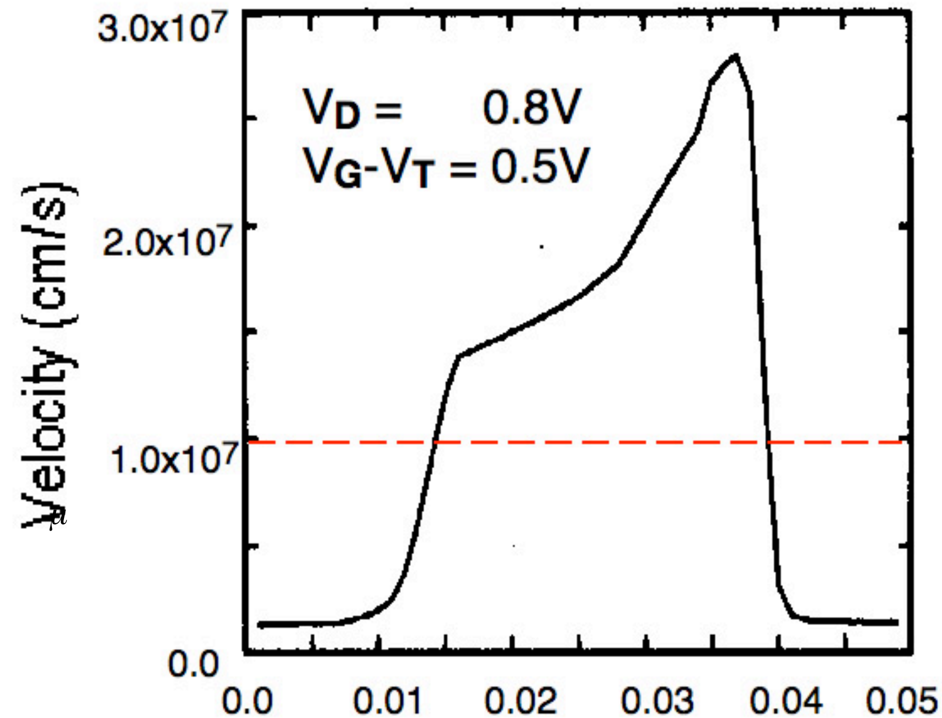
$$I_D = W\,C_{ox}(V_{GS} - V_T)\upsilon_{sat}$$

$$I_D = W\,C_{ox}\,\upsilon_{sat}(V_{GS} - V_T)$$

MOSFET IV: velocity overshoot



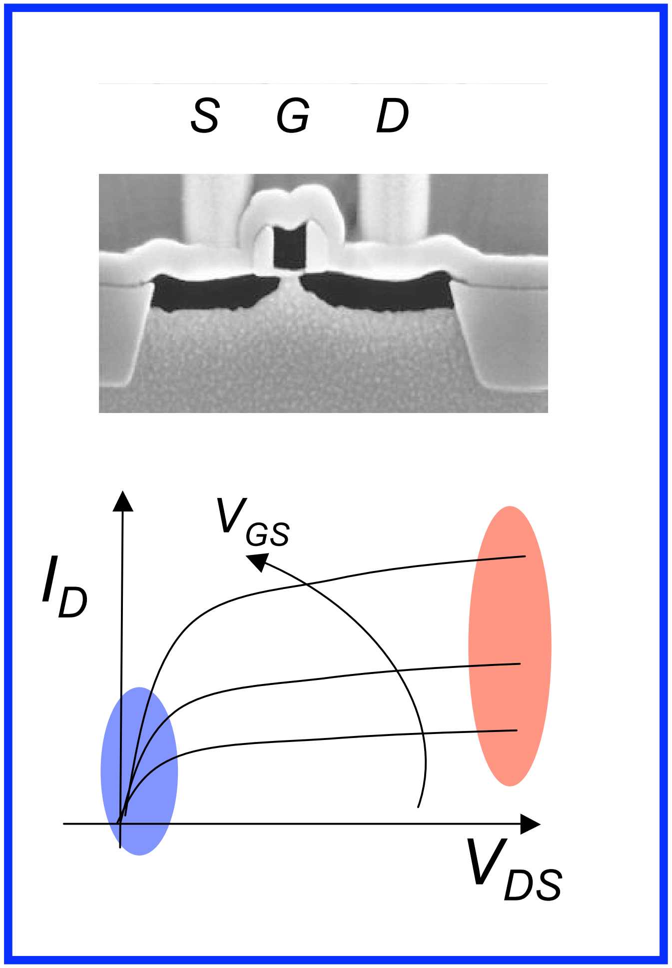
Position along Channel (μm)



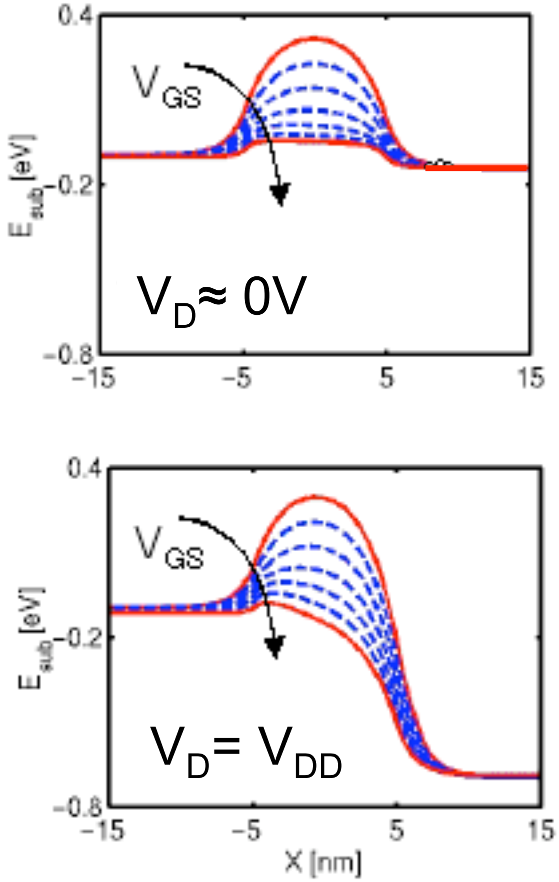
Position along Channel (mm)

Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

the MOSFET as a BJT



electron energy vs. position



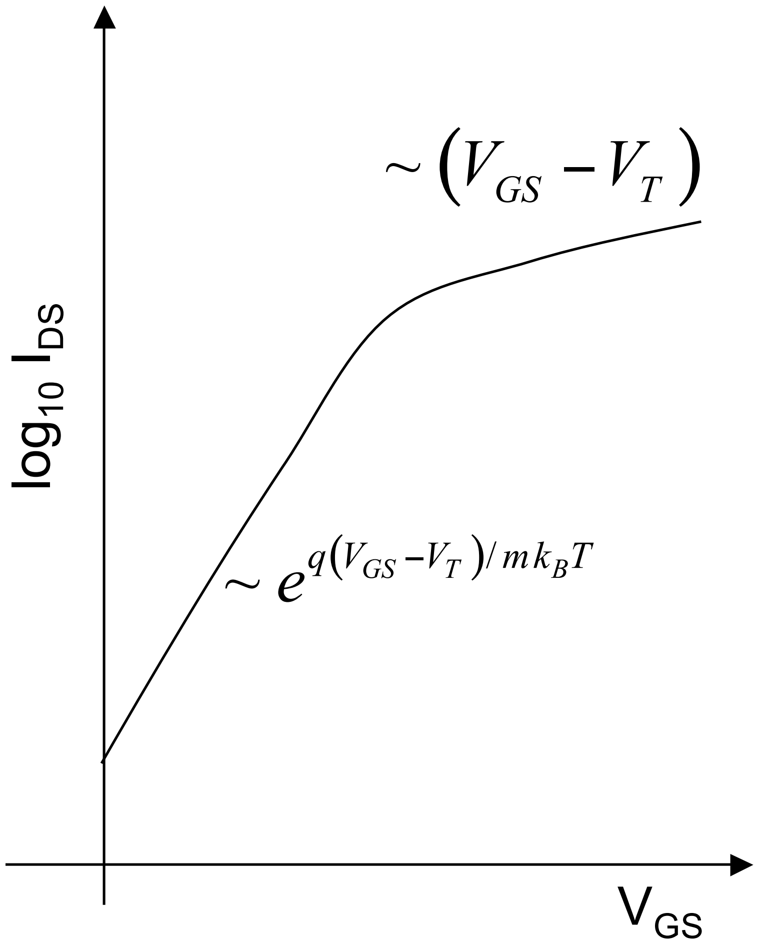
E.O. Johnson, *RCA Review*, **34**, 80, 1973

MOSFET Theory:

$$I_D == \mu_{eff} C_{ox} \left(\frac{W}{L}\right)(m-1)\left(\frac{k_b T}{q}\right)^2 e^{q(V_{GS}-V_T)/mk_BT}\left(1-e^{qV_{DS}/k_BT}\right)$$

eqn. (3.36) on p. 128 of *Fundamentals of Modern VLSI Devices*, Yuan Taur and Tak Ning, Cambridge Univ. Press, 1998.

the MOSFET as a BJT



above threshold:

$$Q_i = C_{ox}(V_{GS} - V_T)$$

$$Q_i \sim e^{q\psi_S/2k_BT}$$

$\Rightarrow$

$$\psi_S \sim \ln(V_{GS} - V_T)$$

$$I_D \sim e^{\psi_S/k_BT} \sim (V_{GS} - V_T)$$

E.O. Johnson, *RCA Review*, **34**, 80, 1973

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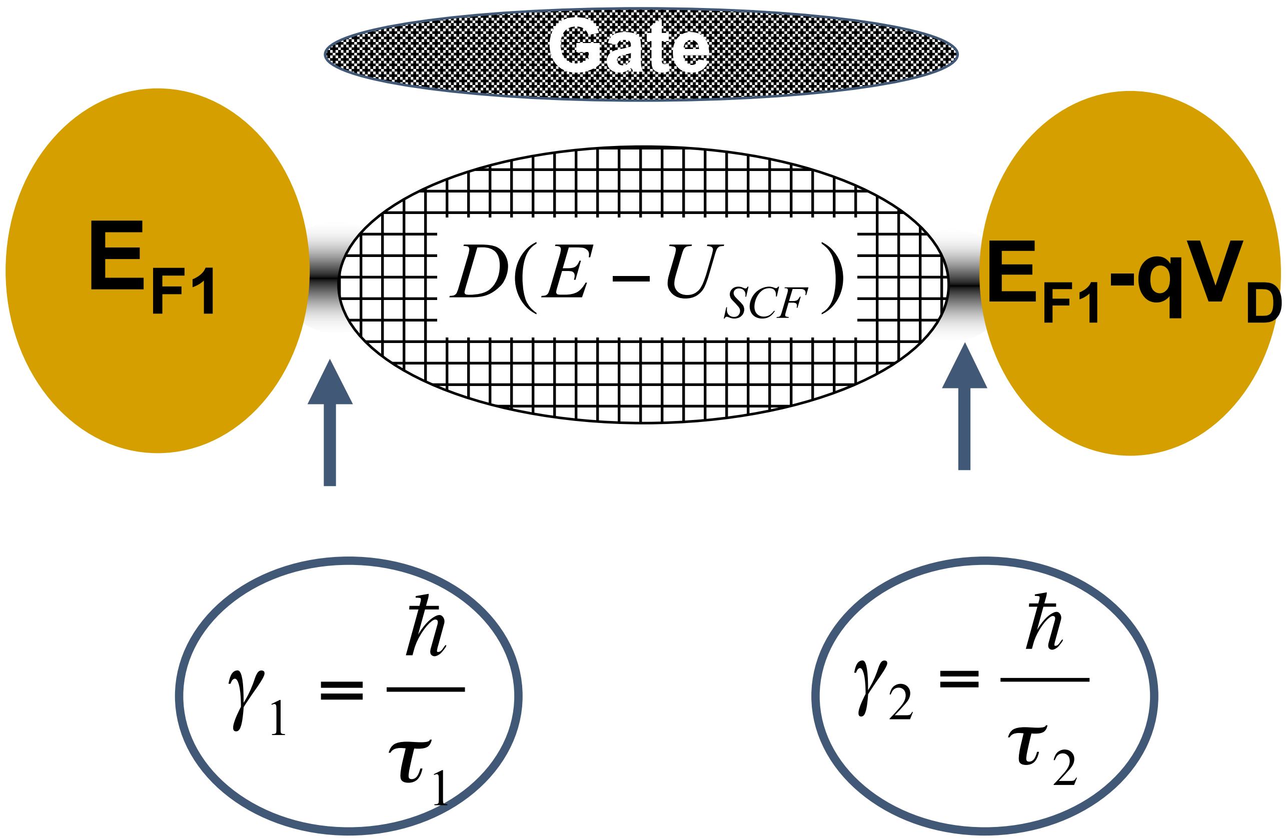
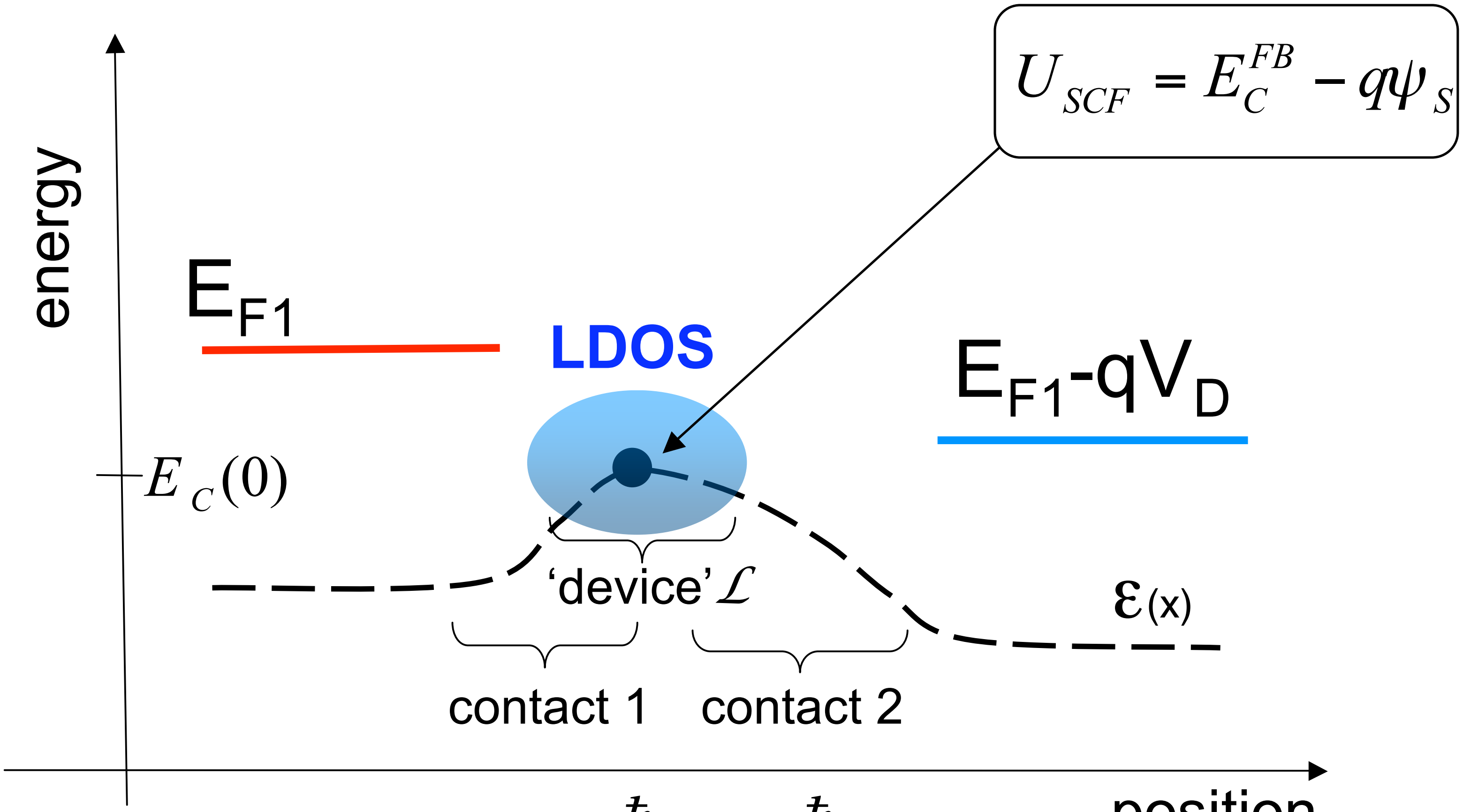
a general view of nano-devices

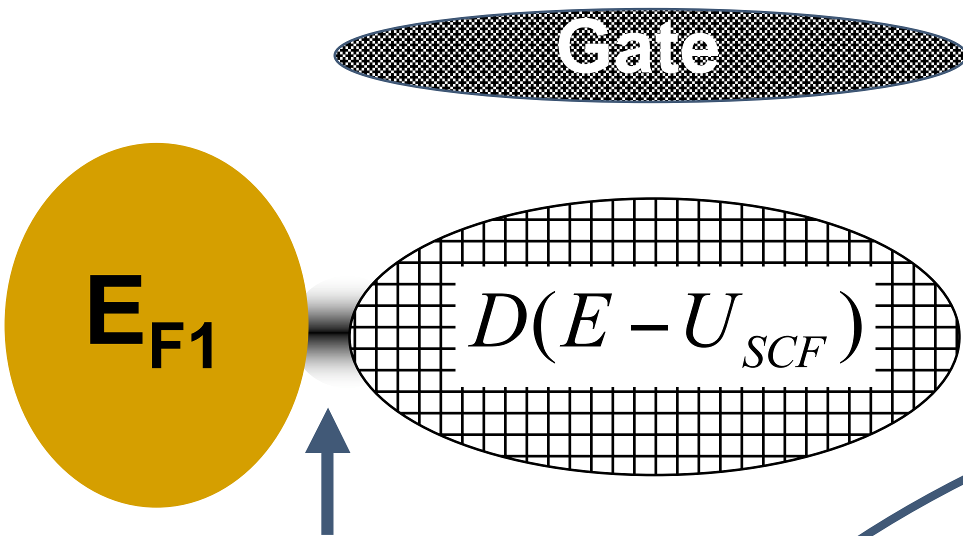


“top of the barrier model”



$$\gamma_1 = \gamma_2 = \frac{\hbar}{\tau} = \frac{\hbar}{\mathcal{L}/\upsilon_x}$$

filling states from the left contact

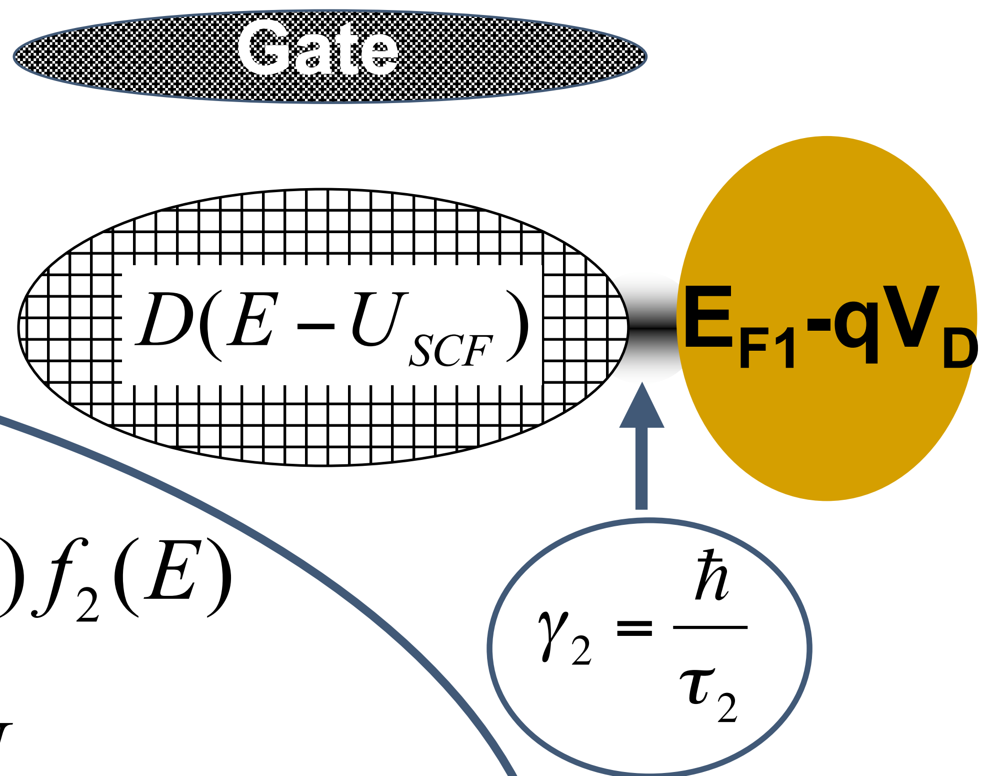


$$\gamma_1 = \frac{\hbar}{\tau_1}$$

$$N_1^0(E) = D(E - U_{SCF}) f_1(E)$$

$$\frac{d\,N(E)}{dt} = \frac{N_1^0(E) - N}{\tau_1}$$

filling states from the right contact



$$N_2^0(E) = D(E - U_{SCF}) f_2(E)$$

$$\frac{dN(E)}{dt} = \frac{N_2^0(E) - N}{\tau_2}$$

steady-state

$$\frac{dN(E)}{dt} = \frac{N_1^0 - N}{\tau_1} + \frac{N_2^0 - N}{\tau_2} = 0$$

(ballistic)

$$N = \int \left[D_1(E) f_1(E) + D_2(E) f_2(E) \right] dE$$

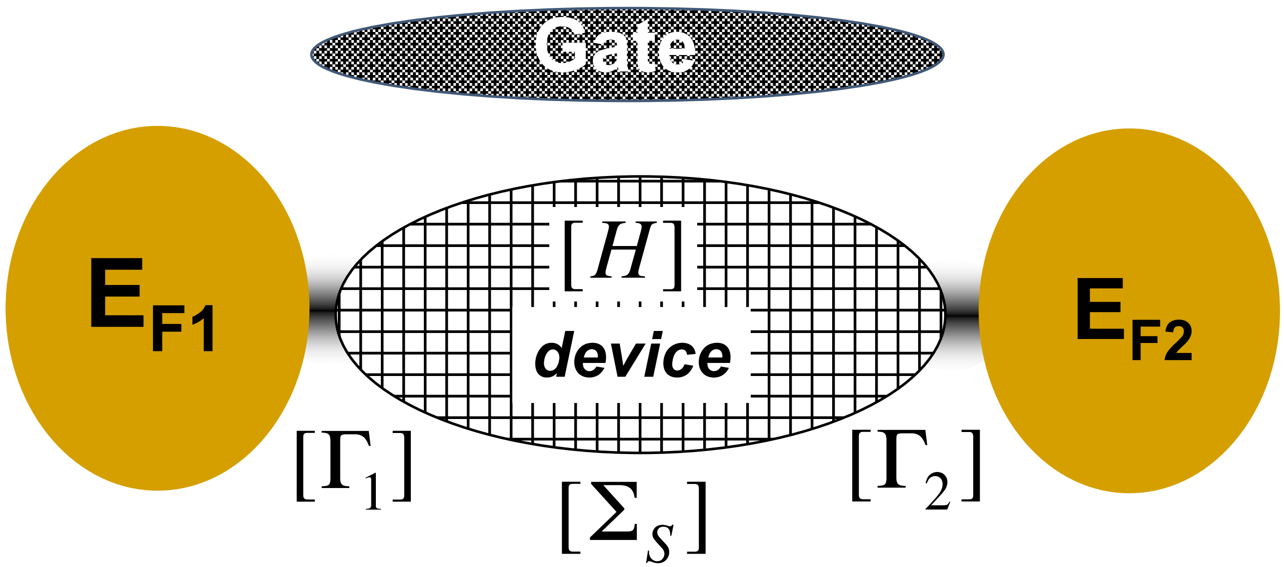
$$D_1(E) \equiv \frac{\tau_2}{\tau_1 + \tau_2} D\left(E - U_{SCF}\right) = \frac{\gamma_1}{\gamma_1 + \gamma_2} D\left(E - U_{SCF}\right)$$

steady-state current

$$I_D = \frac{N_1^0 - N}{\tau_1} = \frac{-\left(N_2^0 - N\right)}{\tau_2}$$

$$I_D = \frac{2q}{h}\int M(E)\left(f_1(E) - f_2(E)\right)dE$$

$$M(E) \equiv \frac{hD(E)}{2\left(\tau_1 + \tau_2\right)} = \pi D(E)\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}$$



Non-equilibrium Green's Function Approach (NEGF)

S. Datta, IEDM Tech. Dig., 2002

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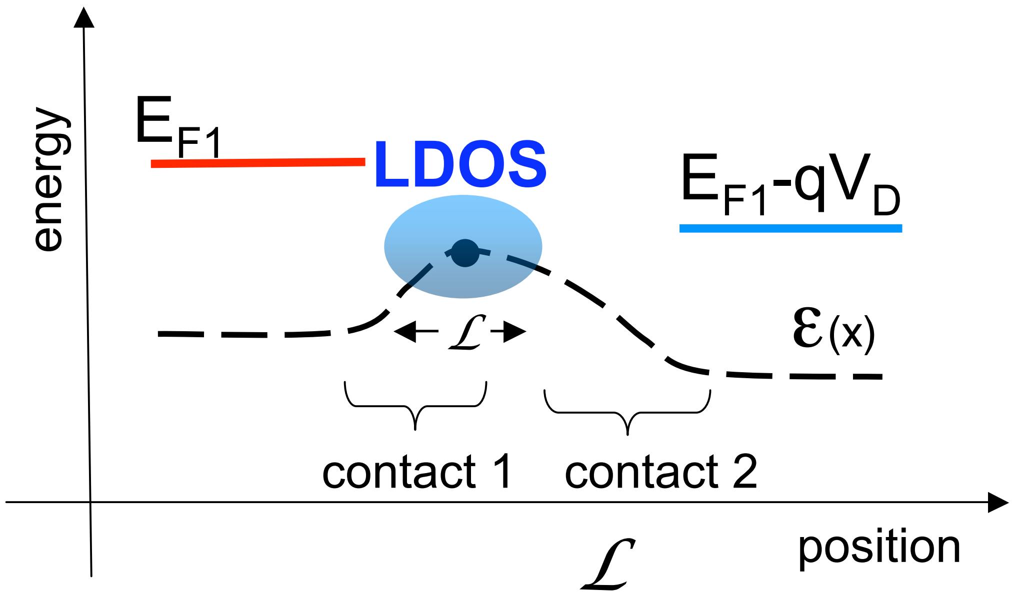
assumptions



$$\tau_1 = \tau_2 = \frac{\mathcal{L}}{\upsilon_x}$$

1) 2D, planar MOSFET

2) 1 subband occupied

$$U_{SCF} \rightarrow E_C = E_C^{FB} - q\psi_S$$

3) parabolic *E*(*k*)

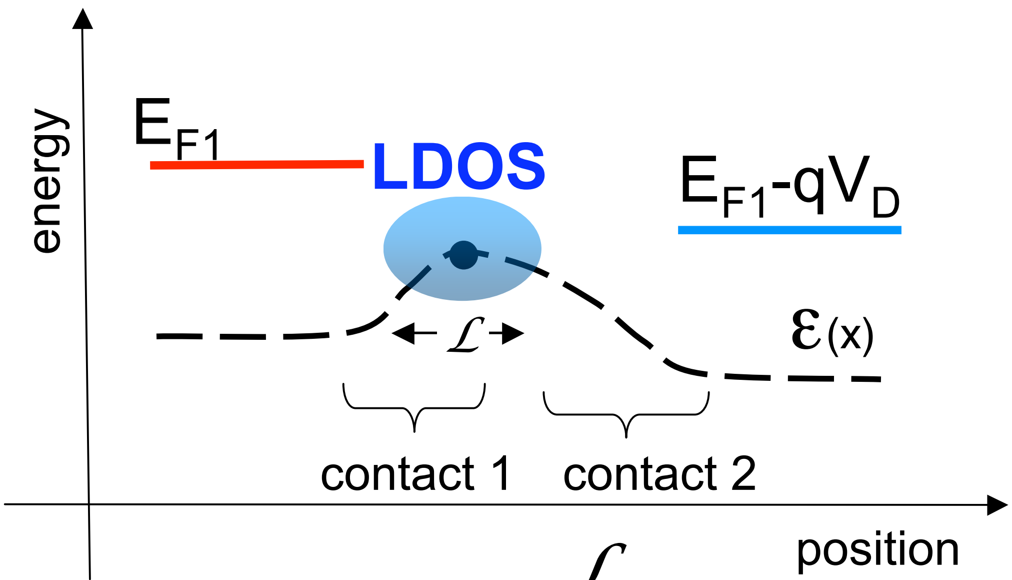
$$D(E) = W\mathcal{L}\frac{m^*}{\pi\hbar^2}\Theta(E - E_C)$$

$$D_1(E) = D_2(E) = D(E)/2$$

$$\langle\tau_1\rangle = \frac{\mathcal{L}}{\langle\upsilon_x\rangle} = \frac{\mathcal{L}}{\sqrt{2E/m^*}\,(2/\pi)}$$

$$M(E) = \frac{W\sqrt{2m^*E}}{\pi\hbar}$$

procedure



$$\tau_1 = \tau_2 = \frac{\mathcal{L}}{\upsilon_x}$$

1) assume a ψ_S
   (sets top of the barrier energy)

2) fill states

$$N = N^+ + N^-$$

3) self-consistent electrostatics

4) evaluate current

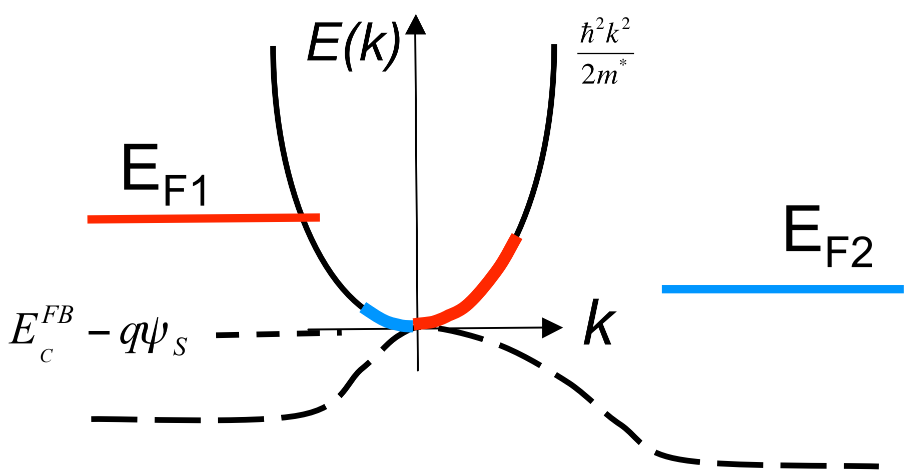
$$I_D = \frac{2q}{h}\int M(E)\big(f_1(E) - f_2(E)\big)dE$$

filling states (ballistic)

N at the top of the barrier depends on:

- V_G (through ψ_S)
- V_D (through E_{F2})

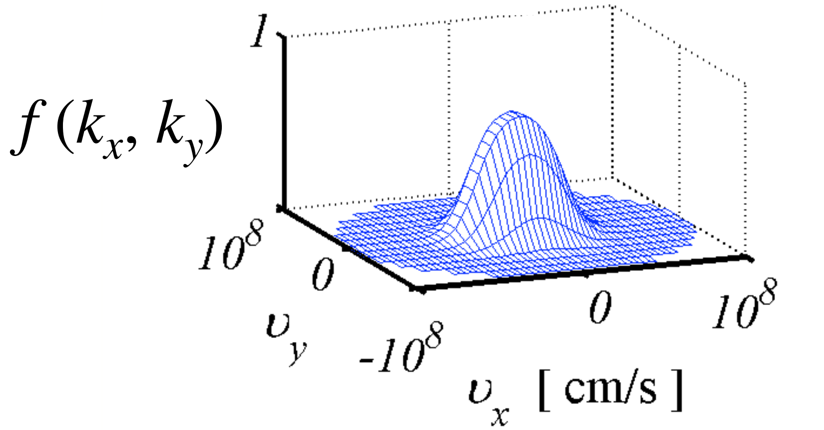
$$N = \frac{N_{2D}}{2} W \mathcal{L} \left[\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2}) \right]$$



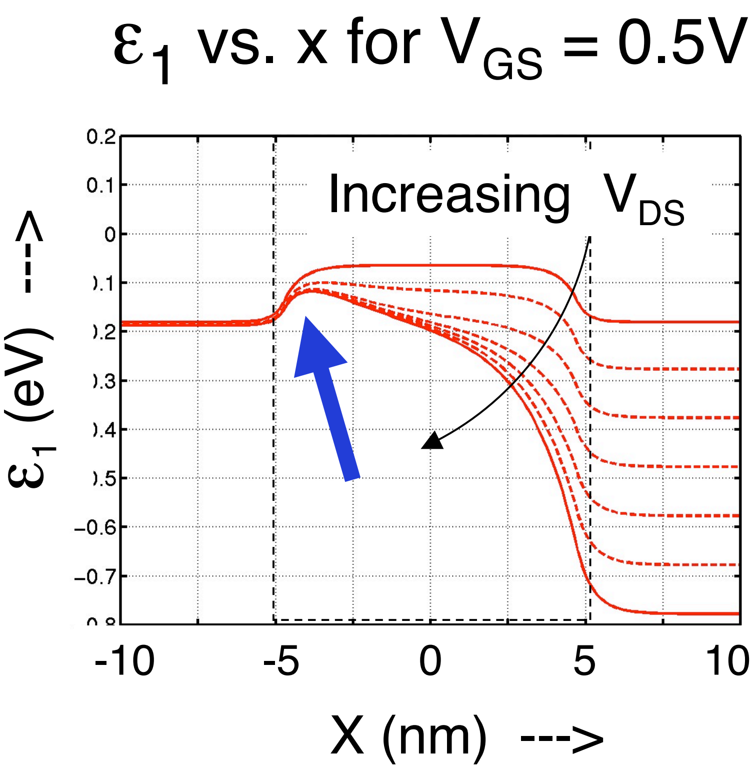
filled states in equilibrium

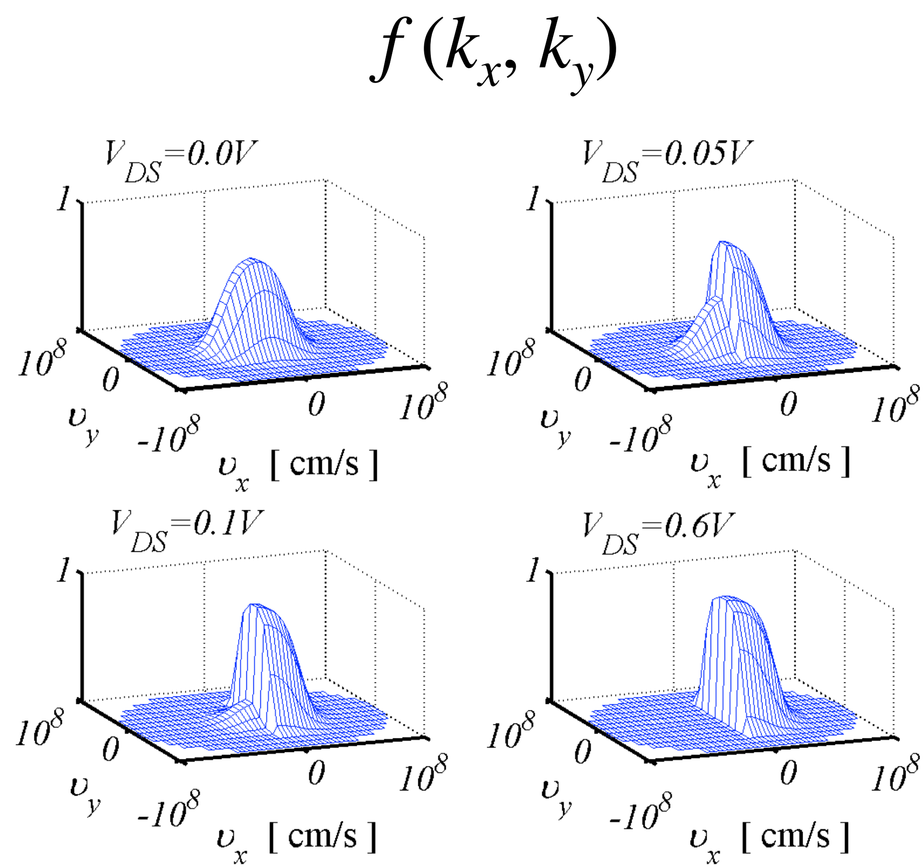
$$f(E) = \frac{1}{1+e^{(E-E_F)/k_B T}} \approx e^{(E_F - E)/k_B T} = e^{(E_F - E_C)/k_B T} \times e^{m^* \upsilon^2 / 2k_B T}$$

$$f(k_x, k_y) \propto e^{\hbar^2 (k_x^2 + k_y^2)/2m^* k_B T}$$



filling states under bias





the ballistic current

$$I_D = \frac{2q}{m^*}\int M(E)\left[f_1(E) - f_2(E)\right]dE$$

$$M(E) = \frac{W\sqrt{2m^*E}}{\pi\hbar}$$

$$I_D = \frac{qN_{2D}}{2} W \upsilon_T \left[\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2})\right]$$

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

$$\tilde{\upsilon}_T = \sqrt{\frac{2k_BT}{\pi m^*}}\left(\frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}\right)$$

alternatively:

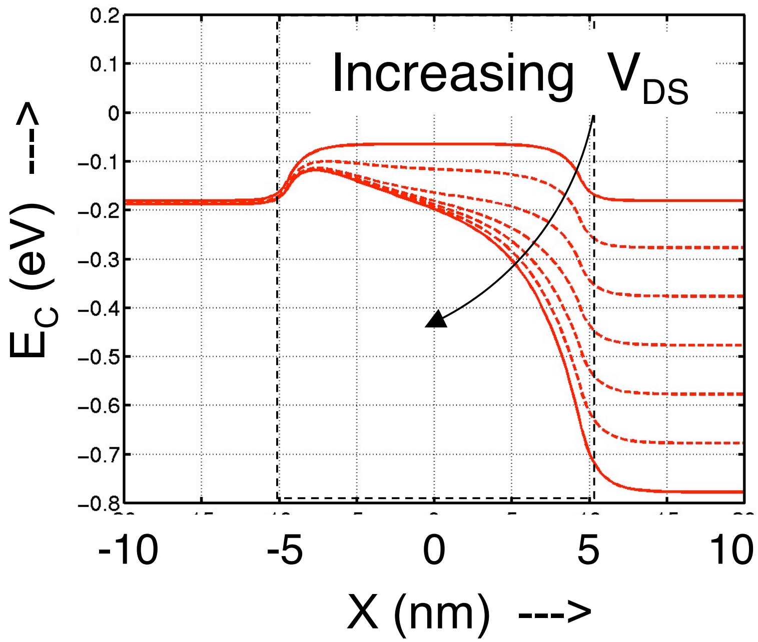
$$I_D \equiv W Q_n \langle \upsilon \rangle$$

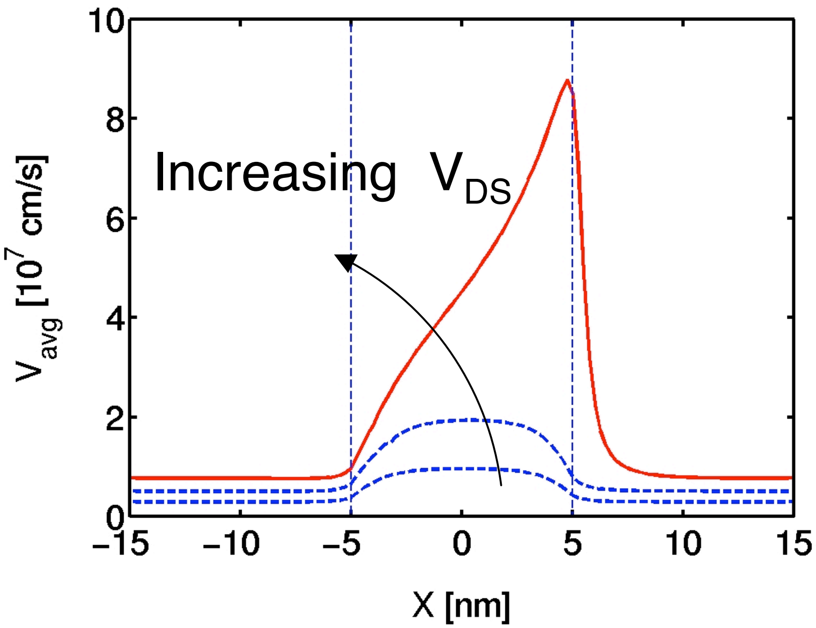
$$Q_n = \frac{N}{W\mathcal{L}}$$

$$\langle \upsilon \rangle = \tilde{\upsilon}_T \left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2})/\mathcal{F}_0(\eta_{F1})}\right]$$

carrier velocity in a ballistic MOSFET

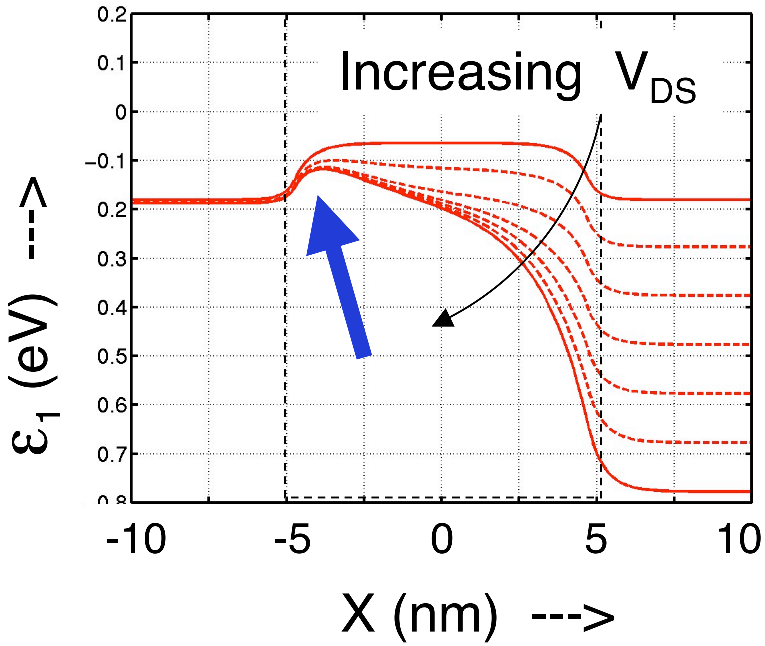
ε_1 vs. x for $V_{GS} = 0.5V$



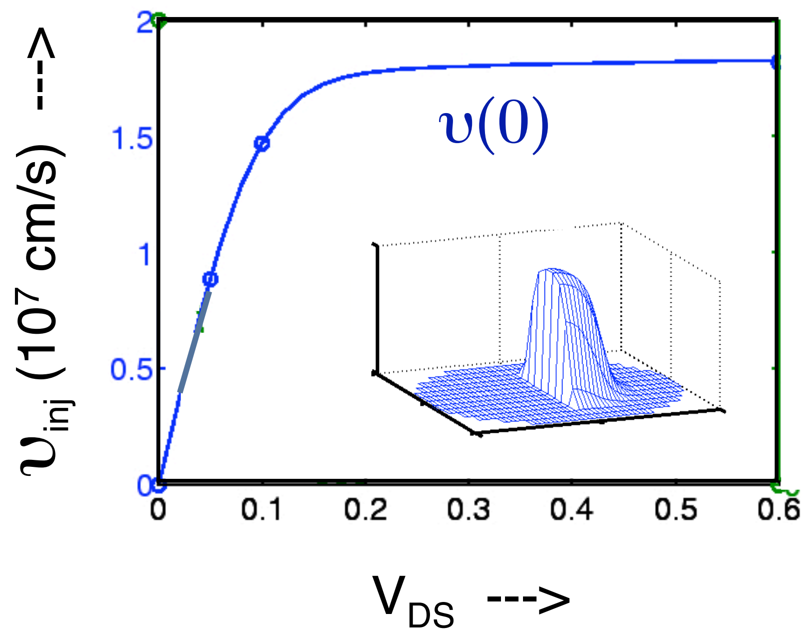


velocity saturation in a ballistic MOSFET

ε_1 vs. x for V_{GS} = 0.5V



$$\upsilon(0) \rightarrow \tilde{\upsilon}_T$$



"injection velocity"

$$\tilde{\upsilon}_T = \sqrt{\frac{2k_BT}{\pi m^*}\left(\frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}\right)}$$

IV of a ballistic MOSFET

Key equations

$$N = \frac{N_{2D}}{2} W \mathcal{L} \left[\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2}) \right]$$

$$I_D = \frac{qN_{2D}}{2} W \upsilon_T \left[\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2}) \right]$$

$$\eta_{F1} = \left(E_{F1} - E_C^{FB} + q\psi_S \right) / k_b T$$

$$\eta_{F2} = \left(E_{F1} - qV_D - E_C^{FB} + q\psi_S \right) / k_b T$$

We must express ψ_S in terms of

V_G
(1D electrostatics)

or

V_G and V_D
(2D electrostatics)

1D MOS electrostatics (above threshold)

Key equations

$$N = \frac{N_{2D}}{2} W \mathcal{L} \left[\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2}) \right] \qquad (1)$$

$$I_D = \frac{q N_{2D}}{2} W \upsilon_T \left[\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2}) \right] \qquad (2)$$

$$C_{ox}\left(V_{GS} - V_T\right) = \frac{qN}{W\mathcal{L}} \qquad (3)$$

equations (1), (2), and (3) give…

1D MOS electrostatics (above threshold)

$$I_{DS} = W C_{ox}(V_{GS} - V_T)\tilde{\upsilon}_T \times \left\{ \frac{1 - \dfrac{\mathcal{F}_{1/2}(\eta_{F1} - qV_{DS}/k_BT)}{\mathcal{F}_{1/2}(\eta_{F1})}}{1 + \dfrac{\mathcal{F}_0(\eta_{F1} - qV_{DS}/k_BT)}{\mathcal{F}_0(\eta_{F1})}} \right\}$$

$$C_{ox}\left(V_G - V_T\right) = \frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F1} - qV_{DS}/k_BT)\right]$$

for non-degenerate statistics:

$$I_{DS} = W C_{ox}(V_{GS} - V_T)\upsilon_T \left\{ \frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}} \right\}$$

the ballistic MOSFET

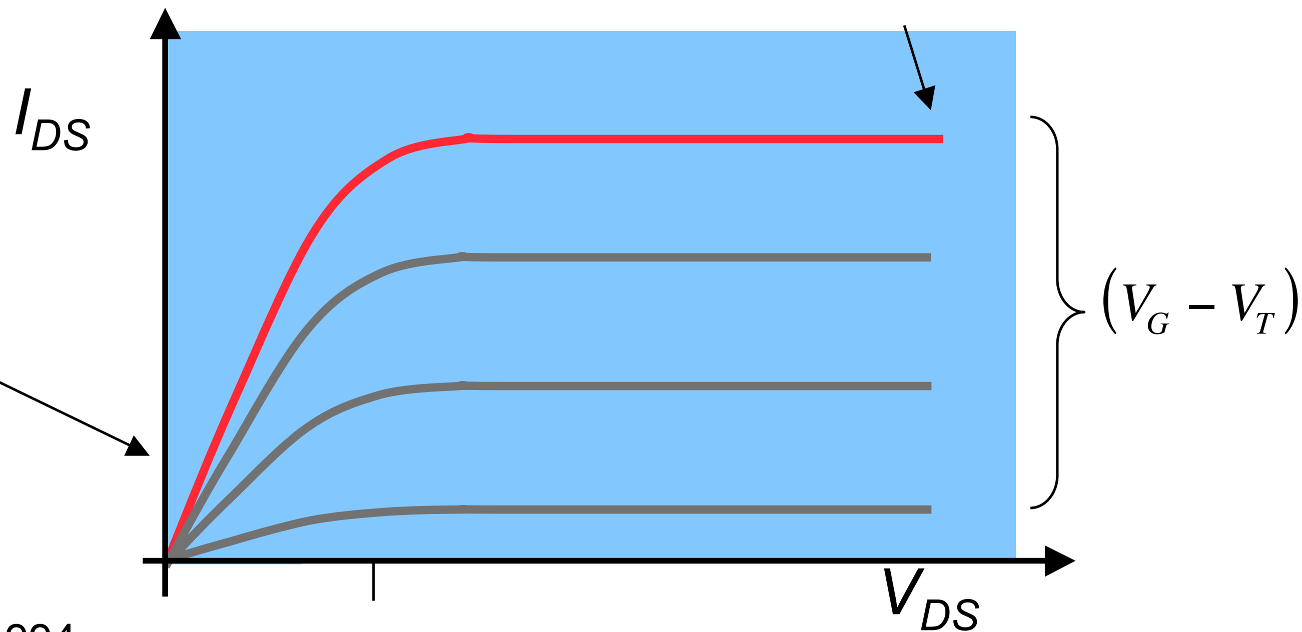
$$I_{DS} = W C_{ox}(V_{GS} - V_T)\tilde{\upsilon}_T \left\{ \frac{1 - \dfrac{\mathcal{F}_{1/2}(\eta_F - U_{DS})}{\mathcal{F}_{1/2}(\eta_F)}}{1 + \dfrac{\mathcal{F}_0(\eta_F - U_{DS})}{\mathcal{F}_0(\eta_F)}} \right\}$$

(ideal electrostatics)

$$I_{DS}(on) = W\ C_{ox}\tilde{\upsilon}_T\left(V_{GS} - V_T\right)$$

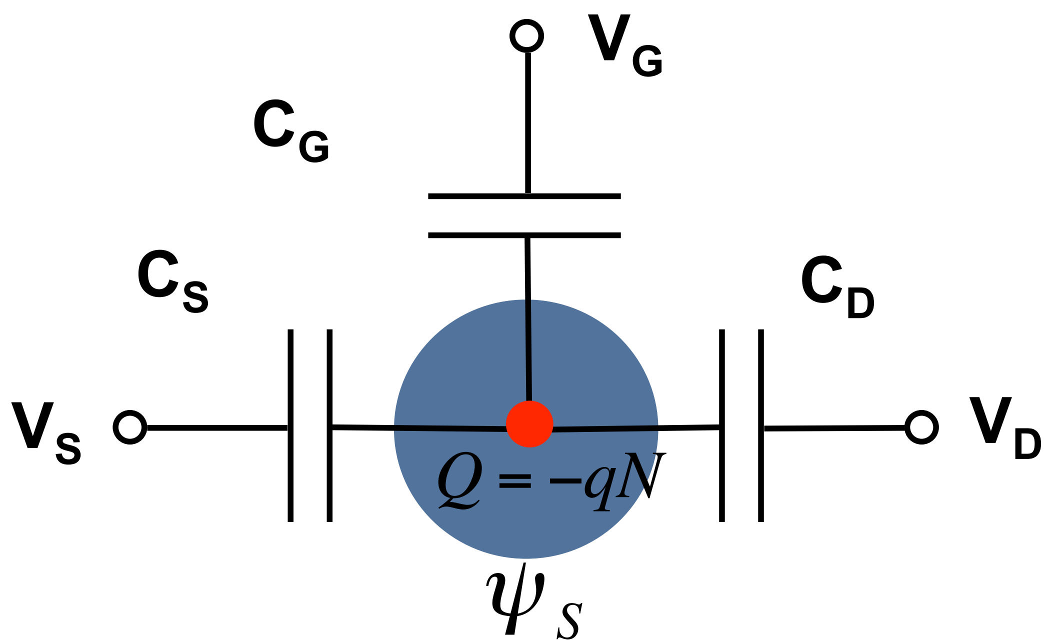
quantum conductance

$$\approx M\frac{2q^2}{h}$$



K. Natori, *JAP*, **76**, 4879, 1994.

electrostatics (subthreshold and 2D)



$$\psi_S = V_G\left(\frac{C_G}{C_\Sigma}\right) + V_D\left(\frac{C_D}{C_\Sigma}\right) + V_S\left(\frac{C_S}{C_\Sigma}\right) - \frac{qN(\psi_S)}{C_\Sigma}$$

procedure

for a given V_G, V_D:

1) *guess* ψ_S

2) *fill states*

3) *compute improved* ψ_S

$$\psi_S = V_G\left(\frac{C_G}{C_\Sigma}\right) + V_D\left(\frac{C_D}{C_\Sigma}\right) + V_S\left(\frac{C_S}{C_\Sigma}\right) - \frac{qN(\psi_S)}{C_\Sigma}$$

4) *iterate between (2) and (3)*

5) *compute current*

$$I_D = \frac{qN_{2D}}{2} W \upsilon_T \left[\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2})\right]$$

6) *select new* V_G, V_D, *and go to 1*

see FETToy at

www.nanohub.org

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the quantum capacitance

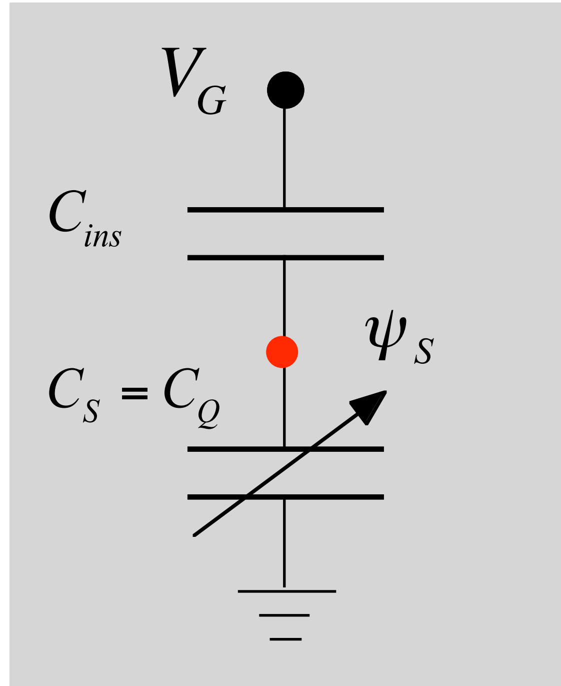
$$Q = C_{Gate}\left(V_G - V_T\right)$$

$$C_{Gate} = \frac{C_{ins} C_Q}{C_{inc} + C_Q}$$

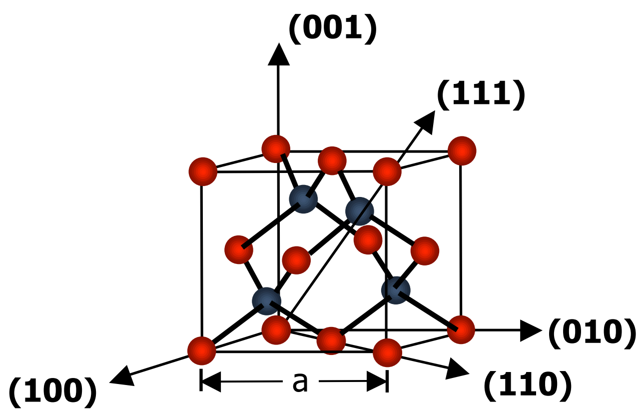
$$C_Q = \frac{\partial\left(-q n_S\right)}{\partial \psi_S} = q^2 \left\langle D_{2D}\left(E_F\right)\right\rangle \sim m^*$$

$$\text{if } C_Q >> C_{ins}, \quad C_{Gate} \rightarrow C_{ins}$$

$$I_D \,\square\, Q \upsilon_{inj}$$



bandstructure effects in nano-MOSFETs



-tight binding model ($sp^3d^5s^*$)
(Boyken, Klimeck, et al.)

-Si, Ge, SiGe, GaAs, InAs, …
(strained or unstrained)
(heterostructure channels)

-bulk, UTB, nanowire MOSFETs

Top-of-the-barrier model

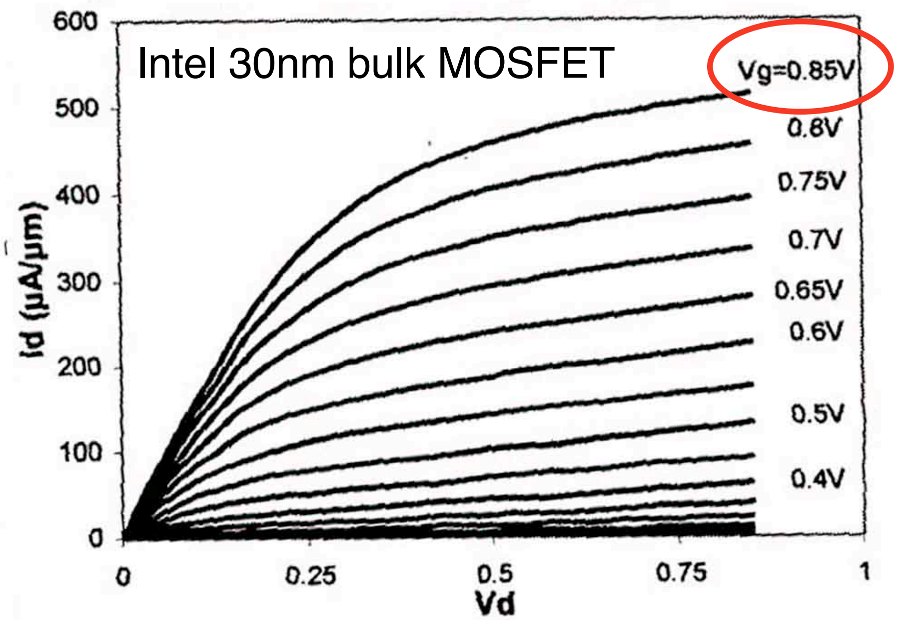
$$E(k) = \frac{\hbar^2 k^2}{2m^*}$$

analytical

$$E(k): \quad \text{tabulated}$$

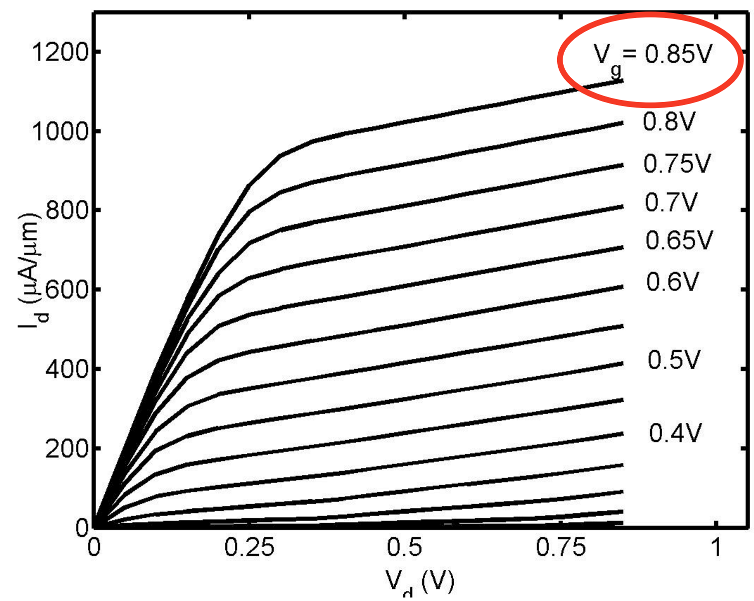
numerical

measured



Chau et al, IEDM Technical Digest, 2000, pp 45 -48

ballistic



MOSFETs operate at ≈ 50% of their ballistic limit

relation to traditional MOSFET theory

$$I_D = \frac{W}{L}\mu_{eff}C_{ox}\left(V_{GS}-V_T\right)V_{DS}$$

low V_{DS}

$$I_D = \frac{W}{2L}\mu_{eff}C_{ox}\left(V_{GS}-V_T\right)^2$$

high V_{DS} (long channel)

ballistic MOSFET

$$I_{DS} = W C_{ox}(V_{GS}-V_T)\tilde{\upsilon}_T \times \left\{ \frac{1-\dfrac{\mathcal{F}_{1/2}(\eta_{F1}-qV_{DS}/k_BT)}{\mathcal{F}_{1/2}(\eta_{F1})}}{1+\dfrac{\mathcal{F}_0(\eta_{F1}-qV_{DS}/k_BT)}{\mathcal{F}_0(\eta_{F1})}} \right\}$$

relation to traditional MOSFET theory

$$I_D = \frac{2q}{h}\int\left[\frac{hD_{2D}(E)}{2\left(\tau_1+\tau_2\right)}\right]\left(f_1(E)-f_2(E)\right)dE$$

ballistic transport: $\langle\tau_1\rangle = \langle\tau_2\rangle = \frac{\mathcal{L}}{\langle\upsilon_x\rangle} = \frac{\mathcal{L}}{\sqrt{2E/m^*}\,(2/\pi)}$

diffusive transport: $\langle\tau_1\rangle = \langle\tau_2\rangle = \frac{\mathcal{L}^2}{2D_{eff}}$

see: “The Ballistic MOSFET,” unpublished notes by M.S. Lundstrom, 2005

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1) A ballistic, "top-of-the barrier" model for the MOSFET is easy to formulate.

2) The ballistic model provides new insights into the physics of nanoscale MOSFETs.

3) Although not comprehensive, the top-of-the-barrier ballistic model should prove useful in exploring new materials and structures for ultimate CMOS.

references

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