Optimum Design of a MEMS Switch

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ABSTRACT

We describe a novel methodology for predicting optimal designs for microelectromechanical devices. The methods are applied towards the optimization of component shapes of a bistable MEMS switch. Small modifications in component shapes lead to a substantial improvement in the device operation.

Keywords: Switch, MEMS, optimization.

1 INTRODUCTION

Optimizing device performance is a critical stage in the design process. The usual approach to optimization consists of several steps: first, a cost function quantifying the device performance is formulated, and the constraints on the device are listed. Next, the device is characterized by a set of parameters. A mathematical model of the device gives the dependence of the cost function on these parameters. The device optimization then consists of finding the design parameters which optimize the cost, subject to the imposed constraints. Typical implementations of this algorithm are computer programs which loop through all possible design parameters until the best design is found.

This algorithm has two serious limitations: (1) First, the numerical cost of applying the algorithm increases exponentially with the number of design parameters. Every new design parameter requires adding another loop to find how the optimum changes. However, in principle every device is described by a continuum of possible design parameters: the shape of every component is a continuous function. The optimal design could have components whose shapes are outside any simple parameterization scheme. (2) Second, a brute force procedure does not focus on the physical mechanism for the property being optimized. For this reason, much of the computational effort is directed towards parts of the device that do not have much effect on the optimization.

In this paper we describe an approach for device optimization, which addresses these issues. The approach is applied to an electrostatically actuated MEMS relay switch. We demonstrate that the optimization predicts shapes of the components of the relay switch which lead to substantial design improvements. For example, an appropriate choice for the shape of the switch leads to an actuation force decreased by a factor of two.

The paper is organized as follows. Section 2 describes the methodology we use for device optimization, in the context of a device made out of cantilever beams. Then we describe the application of these ideas to a MEMS relay switch, with Section 3 describing the optimization of the relay switch. A summary is given in Section 4.

2 OPTIMUM DESIGN ANALYSIS

We start by supposing that a general design concept has been proposed, and describe a methodology for optimizing this design with respect to a class of design parameters. First it is necessary to invent a cost function C that quantifies how well the design works. Then one must be able to efficiently compute gradients of the cost function with respect to the design parameters. A gradient search then allows achieving the optimimum design. We describe this in the context of devices made out of cantilever beams (appropriate for the application described herein), though generalizations to other contexts are straightforward. If $I = bh^3/12$ is the moment of inertia (where h is the beam thickness and b its breadth), then the equation for the displacement w of the beam from its unstressed configuration w_0 is

$$(EI(w - w_0)'')'' + Tw'' = f, \qquad (1)$$

where E is the Young's modulus, T is the tension in the beam and f is the external forcing. The boundary conditions on the beam depend on how it is attached to supports and on any forcing at the ends of the beam. The tension in the beam is either determined by both external compression or extension, and by any compression (extension) of the beam from its rest configuration. The latter results in a tension $T = ES\Delta L/L$, where S is the cross section of the beam, and $\Delta L/L$ is the relative extension of the beam from its rest length L.

A typical cost function C depends both on w and on the free eigenvalues of the beam. (The free eigenvalues T_i and eigenfunctions w_i solve $(EIw_i'')'' = -T_i w_i''$). The problems we consider here involve optimizing C with respect to changes in the shape of the beam (I). We will also consider important experimental constraints,

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such as requiring that the stress in the beam is smaller than some yield stress.

Once the cost function C constraints are defined, optimization proceeds by computing the gradient of C with respect to arbitrary changes in the design variable (I), and then changing I to lower the cost, maintaining necessary constraints. This proceeds iteratively until the gradient vanishes. As usual in a simple gradient search, the only guarantee of this procedure is that the predicted design is a local optimum of the cost function.

For a device with a large number of design parameters, the most expensive part of this algorithm is the computation of the gradient of the cost function with respect to the design parameters. Changing $I \to I + \delta I$ causes the cost function to change $C \to C + \delta C$. A direct calculation of δC requires computing the change in the cost for each changed design parmeters. Since evaluating the cost function *once* requires both solving the above differential equation for w and finding its eigenvalues, this procedure is extremely expensive. A practical implementation of a gradient search algorithm requires efficient calculation of the gradient of $C[w(T_i)]$.

2.1 Eigenvalue dependent cost function

Here, we describe how to efficiently calculate the gradient of a cost function of the eigenvalues alone, $C(\{T_i\})$. Since $\delta C = \sum_{i=1}^{\infty} \frac{dC}{dT_i} \delta T_i$, we need a formula relating the change in the eigenvalues caused by a change $I \to I + \delta I$. This follows directly from equation (1): if $I \to I + \delta I$, then the i^{th} eigenfunction (eigenvalue) changes from $w_i \to w_i + \delta w_i$ and $T_i \to T_i + \delta T_i$. This implies

$$(EI\delta w_i'')'' + T_i^2 \delta w_i'' = -\delta T_i w_i'' - (E\delta I w_i'')''. \qquad (2)$$

For this equation to have a solution, the right hand side of the equation is orthogonal to any null solution of the left hand side. This solvability condition is

$$\delta T_i = E \frac{\int \delta I w_i''^2 \, dx}{\int w_i^2 dx}. \qquad (3)$$

This formula explicitly gives the change in the eigenvalues with respect to an arbitrary change in I. Thus we have an explicit formula for the gradient of δC with respect to δI. The numerical cost of computing this gradient is identical to that for computing the eigenvalues and eigenfunctions. The amount of work is therefore independent of the number of design parameters.

The existence of fast algorithms for computing gradients of cost functions with respect to arbitrary design parameters is an elementary consequence of the variational calculus and is routinely used in optimal control theory [1]–[4]. Applications of such algorithms to design of airplanes has been applied by Jameson [5]; applications to elementary problems in solid mechanics have

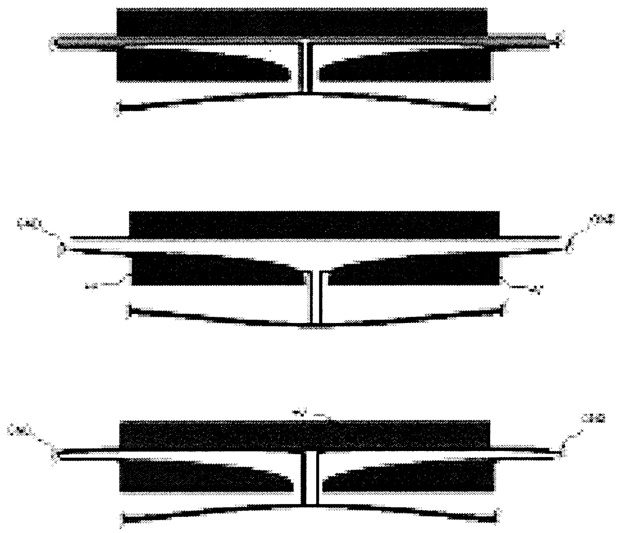

Figure 1: Schematic for the operation of the relay switch. Cantilevered starting zippers (A) initiate the zipping by closing the gap and then allowing the beam to zip closed when the middle electrode is charged (B) causing the switch to close. Turning on the top electrode (C) causes the top starting zippers to engage and the switch opens.

been given by [1], [3], generalizing the solution of Keller for the shape of the strongest column of material of fixed volume. We also note that this optimization procedures differs both practically and conceptually from the so-called topology optimization methods [6].

3 THE RELAY SWITCH

We will focus on our application of these ideas to designing an electrostatically actuated bistable MEMS relay switch, the idea for which is shown in Figure 1. We will first describe the operation of the device and derive the appropriate cost function to achieve the design objective. Then we will apply the above algorithm to deduce an optimal design.

3.1 Overall Design

The figure shows the three stages in the operation of the device: The switch is the lower beam, whose unstressed shape is curved. The vertical crossbars prohibit the center of the beam from twisting. Qiu *et. al.* [7] showed that this makes the switch bistable, with two stable equilibrium states. The device can therefore be used as a mechanical switch. A metallic contact is attached to the bottom of the switch, so that when the beam is pushed from the unstressed equilibrium (Fig 1A) to the second equilibrium (Fig 1B) the switch can open or close (Fig 1C). The great advantage of this switch over other alternatives (latch-lock mechanisms[8], [9] ; hinged multi-segment mech-

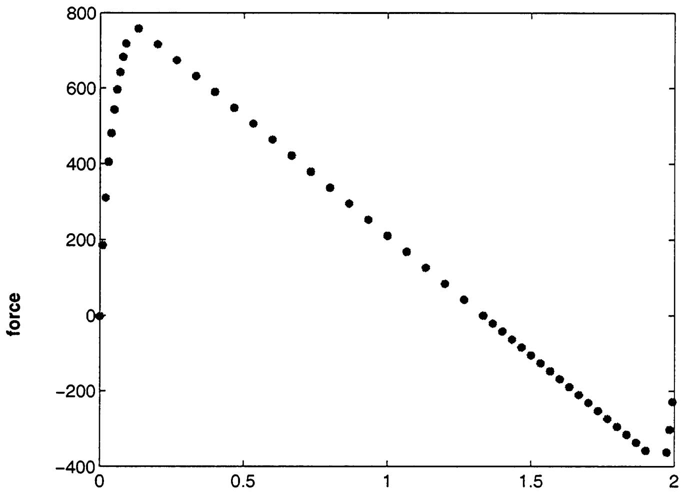

Figure 2: Force displacement characteristics for the switch. The ratio of the maximum force to the minimum force is approximately 2.

anisms[10] ; residual-compressive-stress buckled-beam mechanisms[11]) is that it uses no latches, hinges or residual stress to achieve its bistability.

The device uses an electromechanical zipper mechanism for actuation. The electrodes are coated with a dielectric layer (oxide film). When the middle electrode (Fig 1B) is charged relative to the top electrode, the uppermost beam zips along the electrode, which in turn pushes against the switch. If the electrical force overcomes the mechanical resistance of the switch, the switch closes. When the top electrode is charged (Fig 1C), the switch opens.

3.2 Optimizing the Switch

First we describe the optimization of the switch. The switch is a beam obeying tequation (1), with length L. Before actuation, the beam shape is $w_0 = d(1 - cos(2\pi x/L))$, as assumed by Qiu *et. al.* ; a force applied to the center causes the beam to flip to another stable equilibrium position. The beam is clamped on both ends and the tension T is determined by how much the beam is stretched from its initial shape w_0. Qiu *et. al.* chose a uniform thickness beam; the force-displacement curve for this design is plotted in Fig. 2.

The displacement is defined to be the distance the center of the beam has moved from its initial (unstressed) position. The force displacement curve has several distinguishing features: first, the switch is bistable, having two equilibrium positions. Second, the force required to push the switch between the two equilibria f_{push} is *twice* that required for the reverse transition f_{pop}, yet it is f_{pop} that directly relates to the contact force.

We want to find the shape of the switch which optimizes this force-displacement curve. The efficiency of the device is determined by the force ratio $R = f_{push}/f_{pop}$. A typical application requires that f_{pop} is above a threshold (e.g., closing the relay switch requires $f_{pop} > 50$mN); Simultaneously the force f_{push} should be minimized for easy actuation. Thus, an optimal device minimizes R. Moreover we have the constraints that (a) the structure should be as small as possible; and (b) the yield stress constraint for silicon cannot be violated.

To find the optimal structure it is first necessary to understand which features of the beam shape determine the force displacement characteristics. It turns out that these are almost completely determined by the properties of the three equilibrium states. Besides the unstressed stable equilibrium $w = w_0$, a straightforward calculation shows the other stable equilibrium has $w = -w_0 + O((h/d)^2)$, and the tension in the beam is $T = \sqrt{2}T_0$, where T_0 is lowest eigenvalue of the beam.

The third equilibrium is the unstable *transition state*, which controls the path for passing from equilibrium (i) to (ii); in the transition state the solution is of the form $w = \Gamma w_0 + A w_2$, where w_2 is the eigenmode of the beam coresponding to T_2. It is straightforward to show from equation (1) that $\Gamma = (1 - (T_2/T_0)^2)^{-1}$, and A is determined from the equation relating the tension in the beam to how much it is stretched.

For a beam with uniform thickness, $T_0 = 2\pi, T_2 = 4\pi$ so that $\Gamma = -1/3$. Thus the displacement in the center of the beam at the transition state is $\Delta_t = 1-\Gamma = -4/3$, as observed in Fig 2. Fig 2 also demonstrates that the force is rougly linear through the transition state. This can be calculated by applying a small force to the center of the beam in the transition state. It can be shown that the force displacement relation near the transition state is given approximately by:

$$f = 4T_2^2(\Delta_t - \Delta), \tag{4}$$

where T_2 is the second free eigenvalue of the beam. Δ_t is the location of the transition state. If we then assume (as suggested by the simulation in Figure 2[1]) that the linear regime around the transition state extrapolates to both of the stable equilibrium states, equation (4) then predicts that $f_{push} = 16/3(4\pi)^2 \approx 842$ and $f_{pop} = -8/3(4\pi)^2 \approx -421$ in excellent agreement with the simulation of Fig. 2. This then gives a simple formula for the force ratio, valid for arbitrary beam shape: $R = f_{push}/f_{pop} = \Delta_t/(\Delta_t - 2)$.

We now can perform the optimization. The force ratio tends to unity as $T_2/T_0 \to \infty$. In turn, the eigenvalues depend on the beam shape. We therefore seek a beam shape $I(x)$ which maximizes T_2/T_0 while maintaining the strain constraint. The algorithm derived

[1]A more complete analysis shows that this assumption is accurate; when the beam thickness is not uniform the force displacement curve is no longer perfectly linear, but the corrections to linearity are small.

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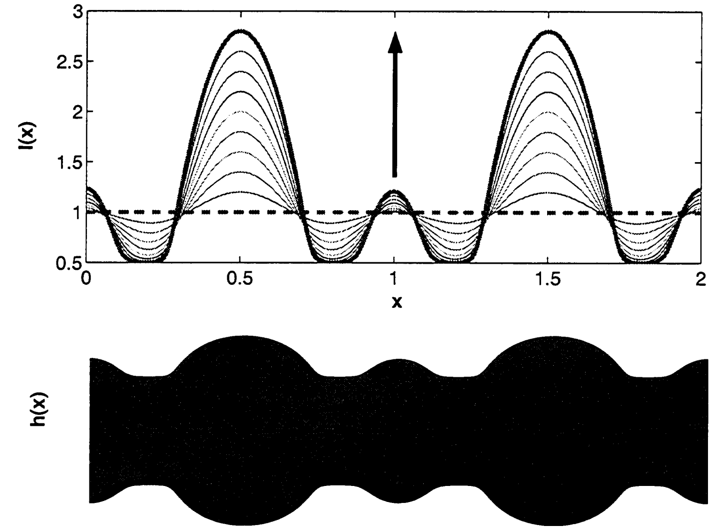

Figure 3: Top: Evolution of the moment of inertia of the beam during the optimization. The initial $I(x)$ is uniform (thick dashed line); with increasing T_2/T_0 the beam becomes modulated. The thick solid line is the final $I(x)$. Bottom: Beam shape corresponding to the optimal $I(x)$. The beam thickness is exaggerated relative to its length by $\approx 10^3$.

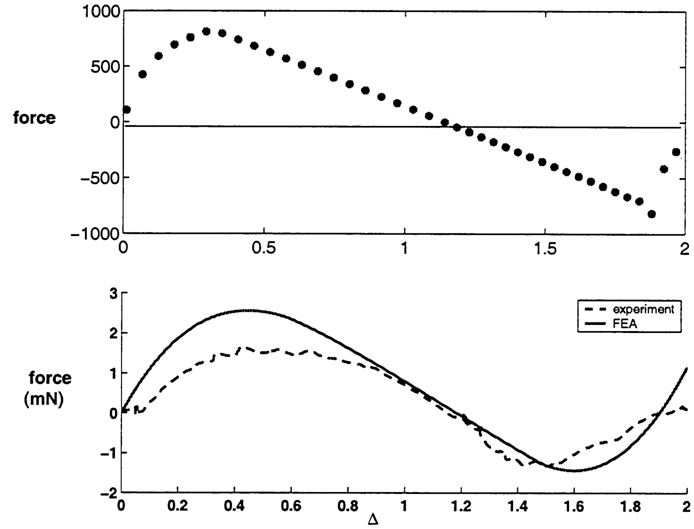

Figure 4: Top: Force displacement characteristics for the optimized structure. The force ratio $R \approx 1$. Bottom: Experiments and finite element analysis of a fabricated double beam of length 9mm, minimum thickness 20μm, and $d = 200\mu$m. The optimization analysis designed this structure to have $R \approx 1.6$, as observed.

above allows us to accomplish this, the results of which are shown in Fig 3-4. Figure 3 shows the evolution of $I(x)$ during the optimization; the bottom part of the figure shows the optimal beam shape. A modest tapering substantially changes the force ratio. The strain constraint is enforced by imposing a minimum beam thickness.

The force displacement characteristics of this device are shown in Fig 4. The force ratio is approximately unity in the optimized structure. The bottom part of Fig. 4 shows experimental data and finite element analysis for a fabricated structure on a silicon wafer, designed to have $R \approx 1.6$. The structure has a force ratio close to the design specifications.

4 SUMMARY

In this paper we have described a general approach to the optimal design of microelectromechanical devices, and the application of this approach to designing the shape of a relay switch. The method starts with a mathematical analysis of the device, identifying the features of the device which allow it to operate most efficiently; in this case, the eigenvalue ratio controls the force ratio of the switch. Then we applied a fast method for computing the gradient of the cost function suggested by this analysis. Because the computational work in computing this gradient is independent of the complexity of the device, we believe that this approach holds great potential for device optimization. In a subsequent manuscript we will describe the application of these ideas to optimizing the actuator for this device.

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