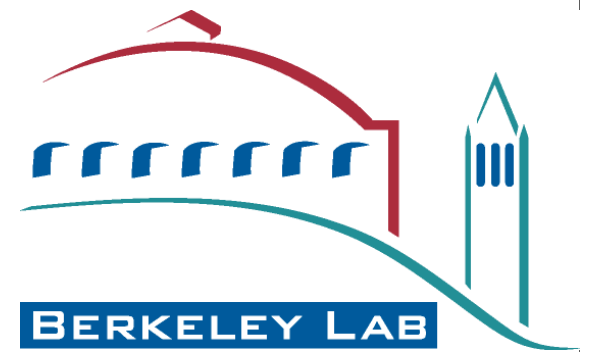


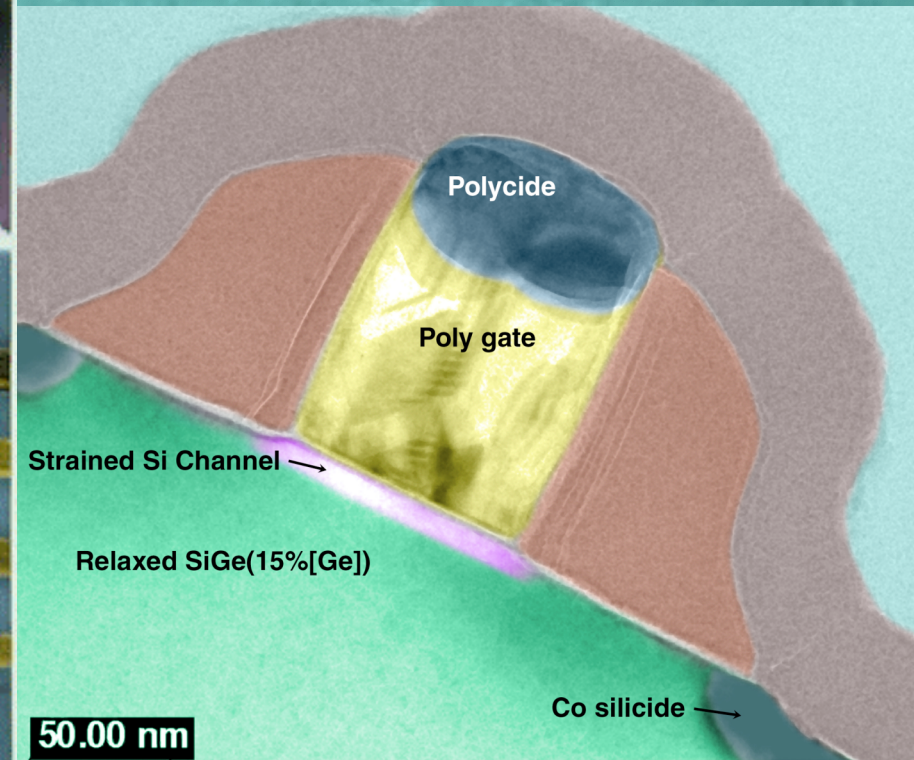
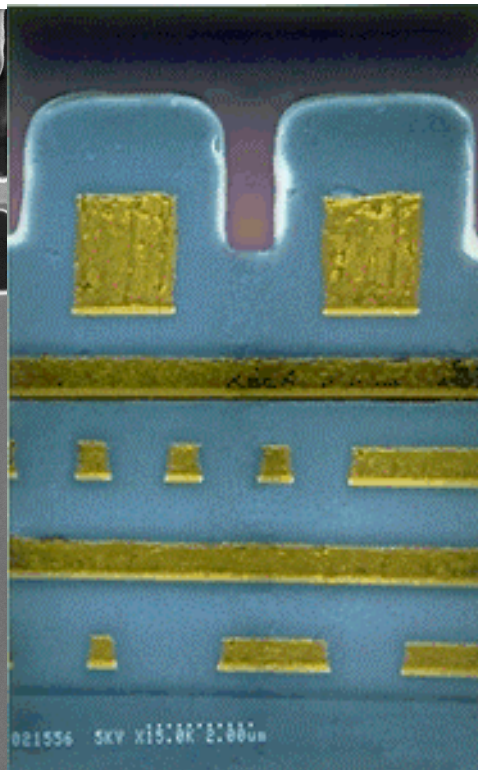
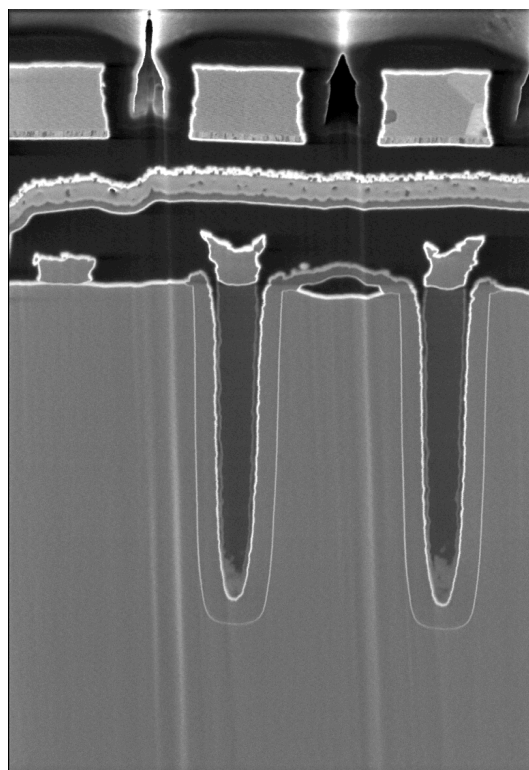
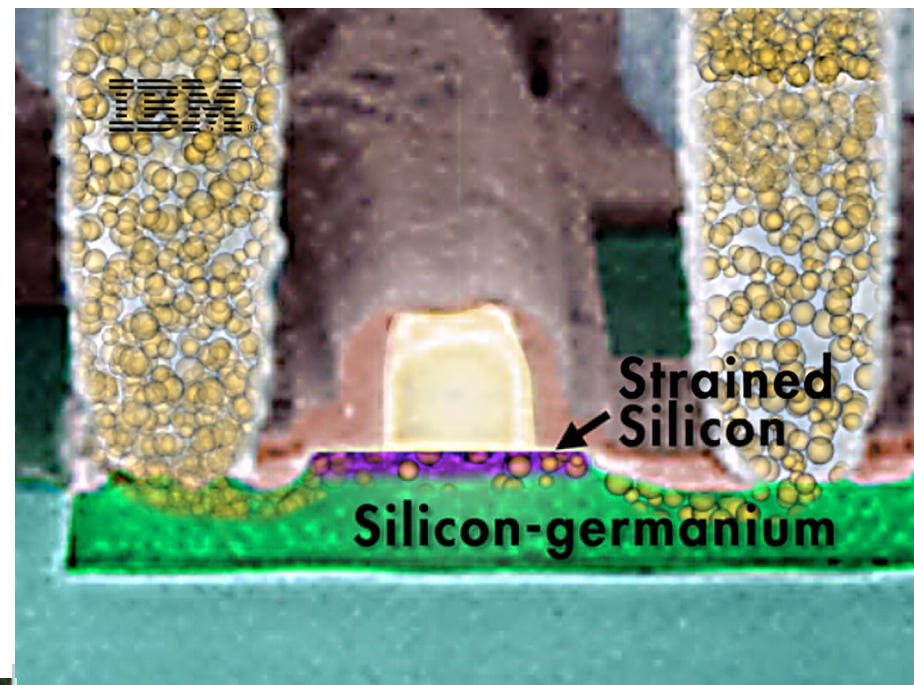
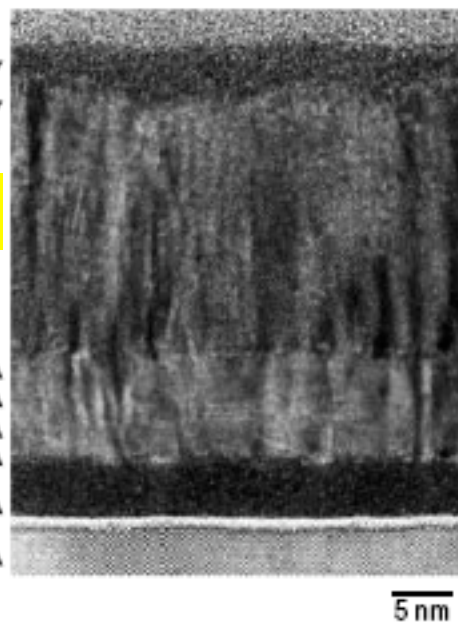
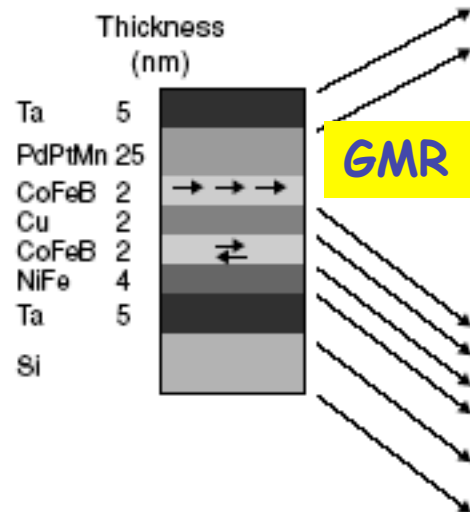
# Heat Transport in Nanostructured Materials & Devices

**Arun Majumdar**

Department of Mechanical Engineering  
University of California, Berkeley

Materials Sciences Division  
Lawrence Berkeley National Laboratory

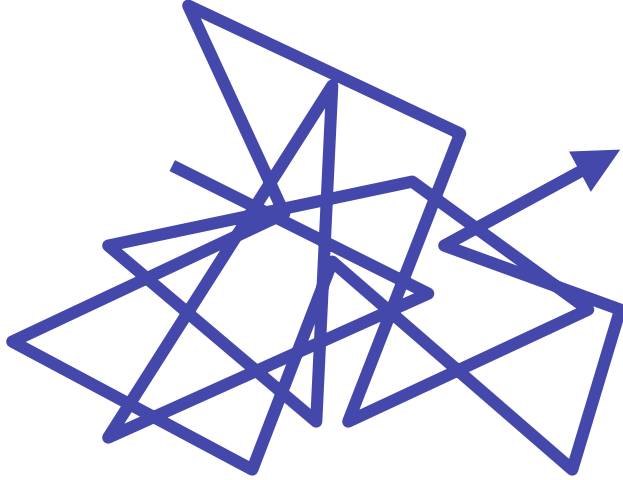




# Overview

- Brief history of transport phenomena
- Characteristic length scales of heat transport
- Spectral nature of heat conduction
- Applications
- Summary

# Heat Conduction



J. Fourier(1768-1830)

## Fourier Law

$$\mathbf{q} = -k\nabla T$$

## Kinetic Theory

$$k = C v \Lambda / 3$$

Specific Heat = C

## Brownian Motion or Diffusion

Mean Free Path =  $\Lambda$

Mean Speed of Propagation =  $v$

## Boltzmann Equation

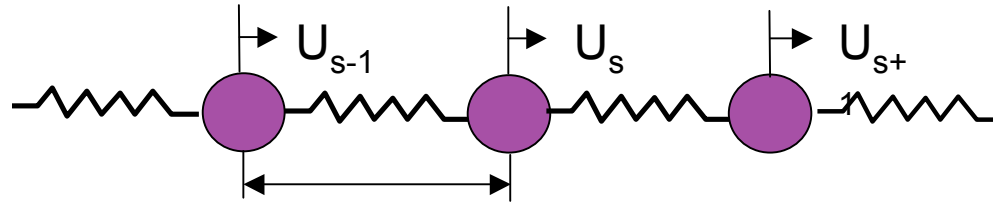
$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f = \left( \frac{\partial f}{\partial t} \right)_{col}$$



L. Boltzmann (1844-1906)

$$k = \frac{1}{3} C v l = \frac{1}{3} \int C(\epsilon) v(\epsilon)^2 \tau(\epsilon) D(\epsilon) d\epsilon$$

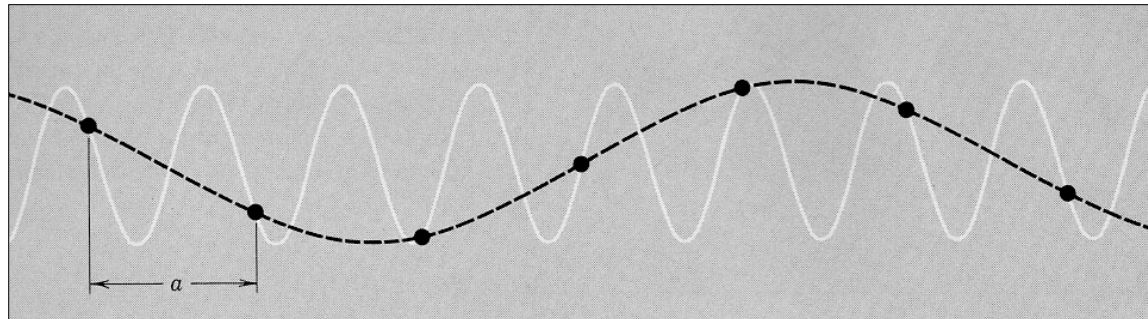
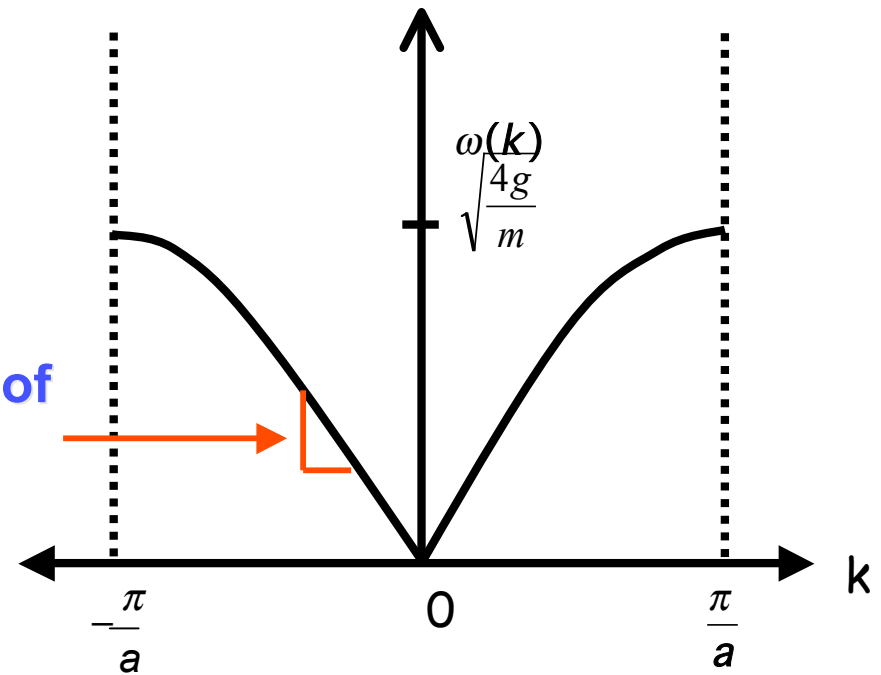
# Phonons



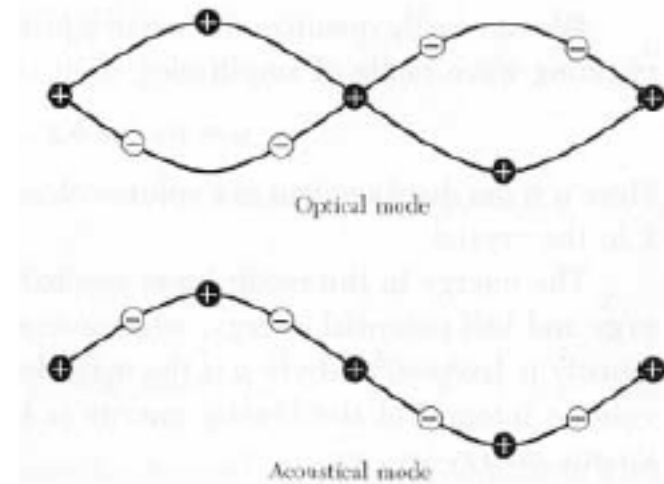
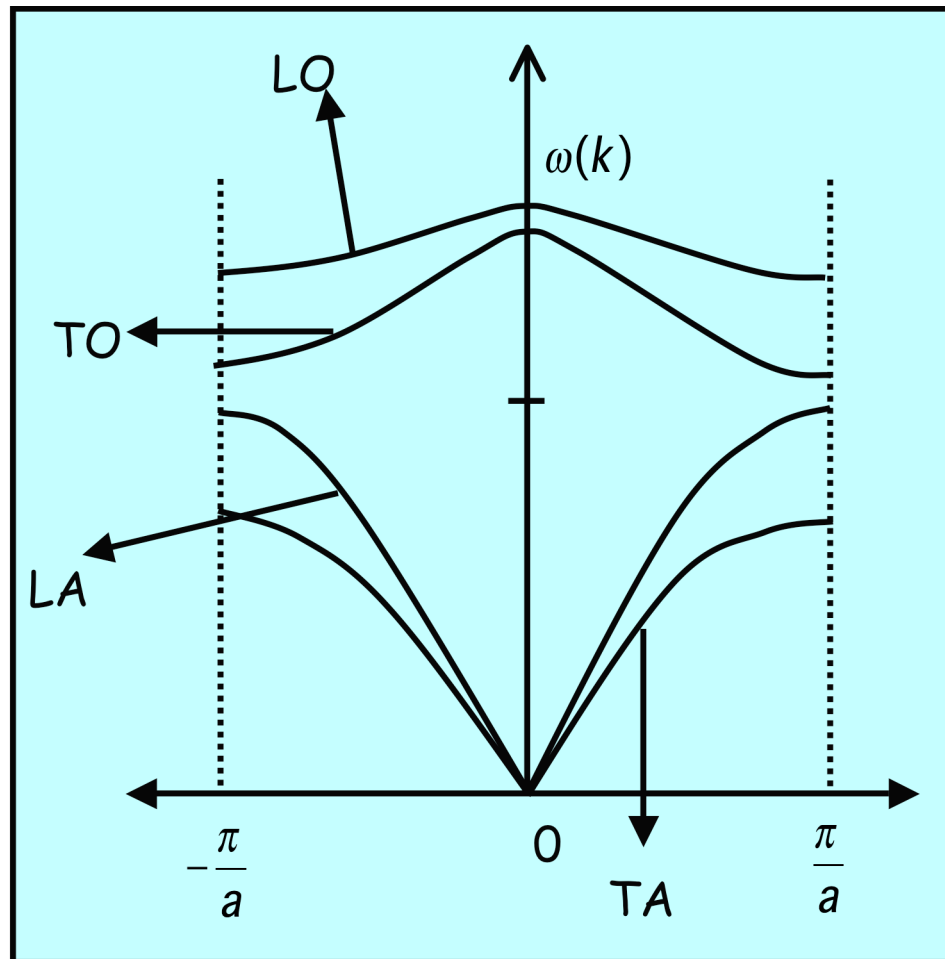
$$\omega(k) = \sqrt{\frac{4g}{m}} \left[ \sin\left(\frac{ka}{2}\right) \right]$$

$$v_g = \frac{d\omega}{dk}$$

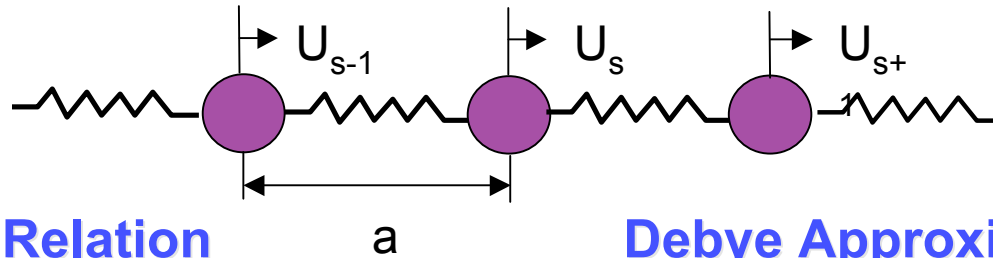
Speed of  
Sound



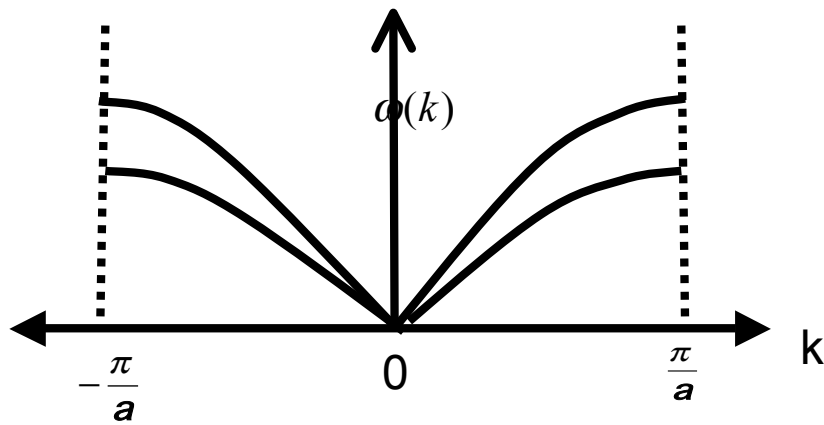
# Phonon Dispersion in a General Lattice



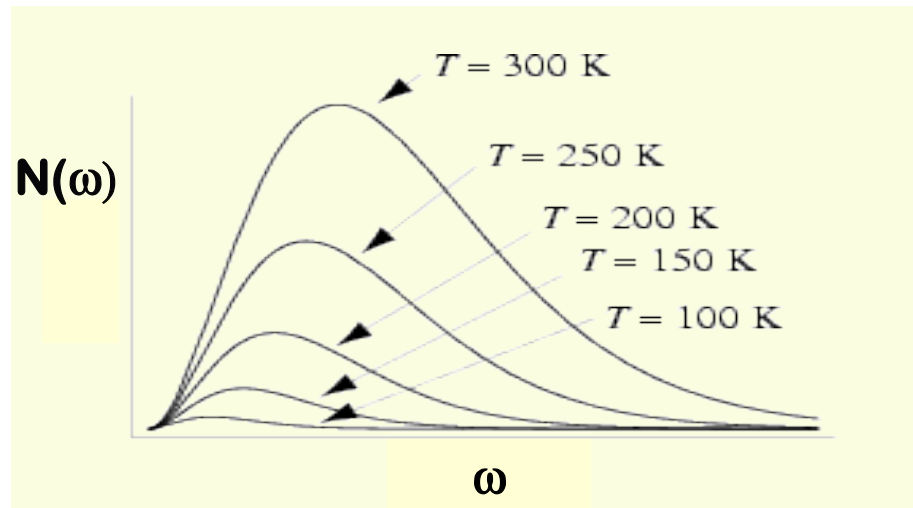
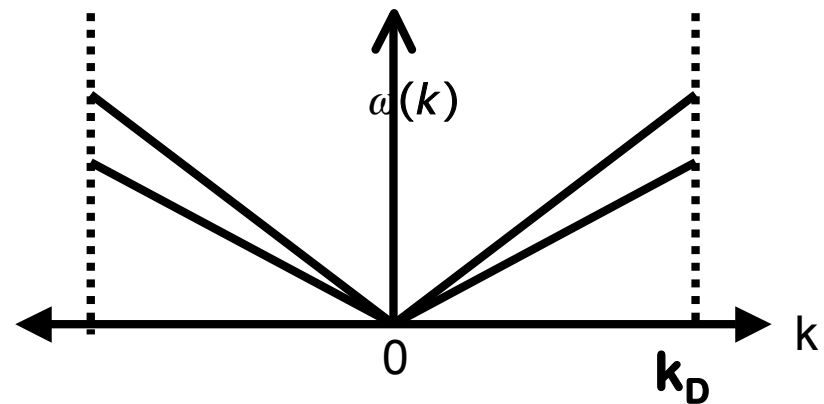
# Phonons



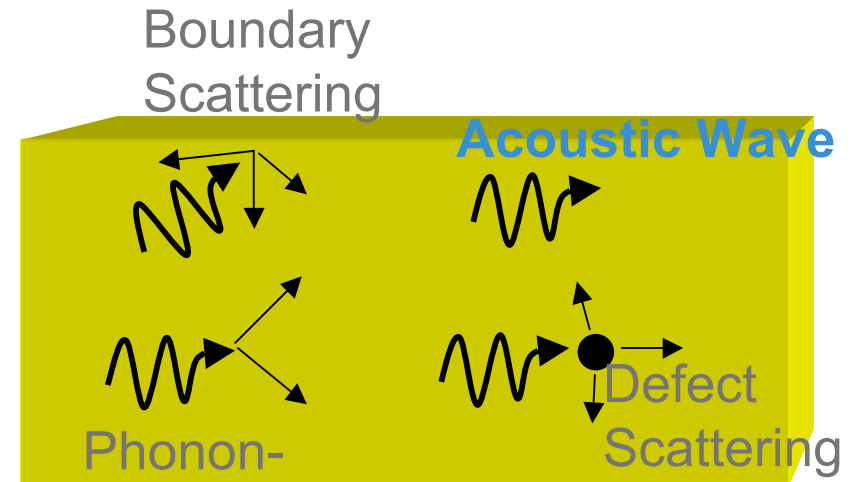
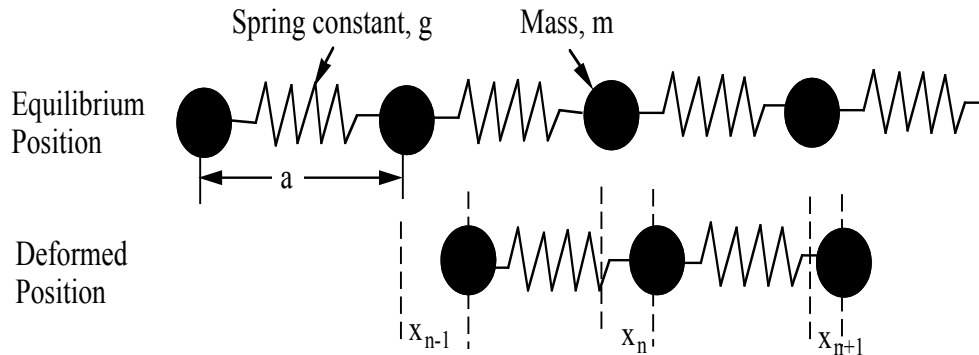
Dispersion Relation



Debye Approximation

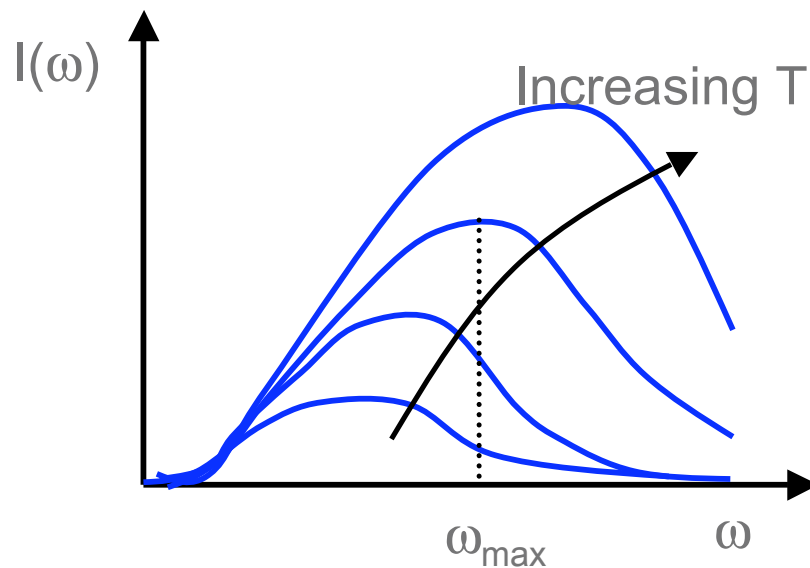


# Physics of Heat Conduction in Nonmetals



Phonon-  
Phonon  
Scattering

Blackbody Phonon Radiation

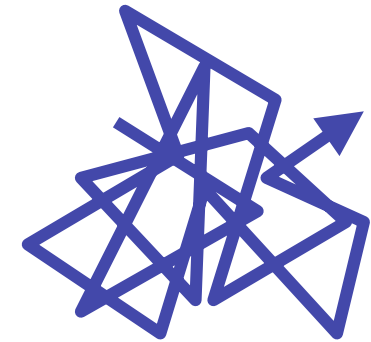


$$\omega_{max} \approx \frac{3k_B}{\hbar} T$$

$$\lambda_{max} T \approx \frac{hc}{3k_B}$$

$$c_{light} = 3 \times 10^8 \text{ m/s}$$

$$c_{sound} \approx 3 - 10 \times 10^3 \text{ m/s}$$

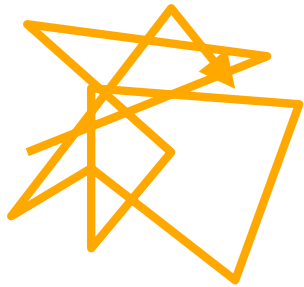


1900

## History of Charge Transport

2006

Uncorrelated  
Scattering  
(Drude)



Free Electron  
Model  
(Sommerfeld)

Electron  
Interference  
(Aharonov-Bohm)

High Mobility  
2D Gases  
(Kroemer, Alferov)

Transistor  
(Bardeen, Brattain,  
Shockley)

Electron  
Tunneling  
(Giaever, Esaki)

Conduction  
Channels  
(Landauer)

Quantum  
Hall Effect/  
Conductance  
(von Klitzing)

GMR  
(Gruenberg, Fert,..)

Spin Dependent  
Scattering

### Technological Impacts

- Integrated Circuits
- Wireless Communication
- Data Storage
- Light Emitting Diodes and Diode Lasers
- .....

 $10^{20}$ 

Low

Glass

Electrical Conductivity/Conductance

High

Cu

**Phonon Optics at Low Temperature (1970s--.)**  
(Narayanamurti, Dynes, et al;  
Wolfe et al.)

**Quantum Phonon Conductance**  
(Schwab, Roukes, 2000)

| Molecular Heterostructures |                                   | Nanowires & Arrays                  |  | Nanotubes                         |
|----------------------------|-----------------------------------|-------------------------------------|--|-----------------------------------|
| Phonon Filtering           | Correlated scattering & diffusion | Uncorrelated scattering & diffusion |  | Waveguiding & ballistic transport |

Ultralow

Polymers

Diamond

Ultrahigh

**Phonon Length Scales (300K)**

Wavelength = 1-10 nm

Mean Free Paths = 10-100 nm

**$10^4$  @300K**

**Thermal Conductivity**

$$k = Cvl/3$$

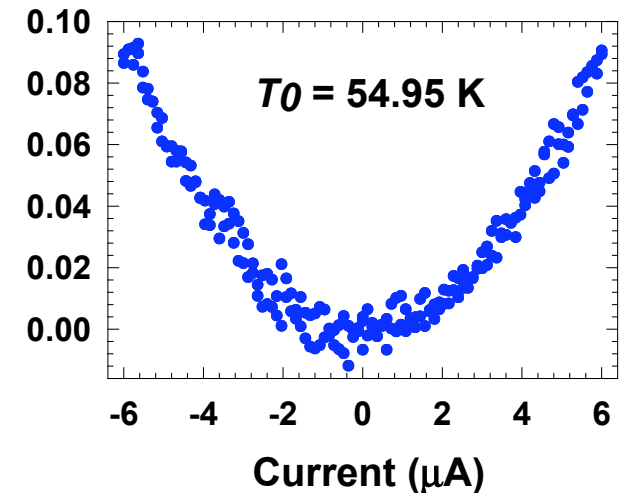
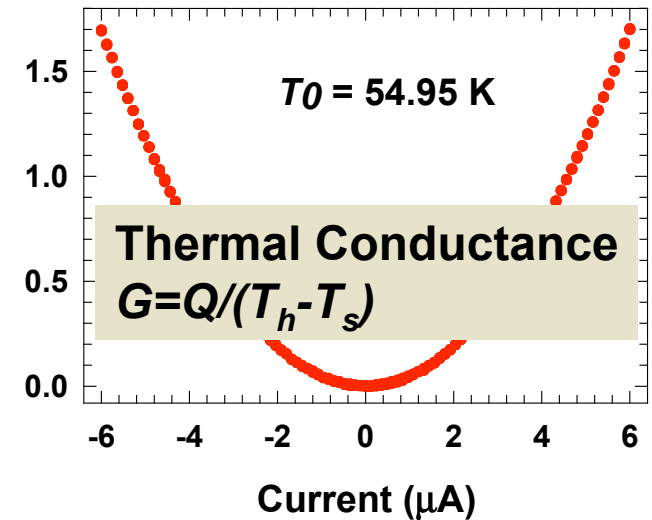
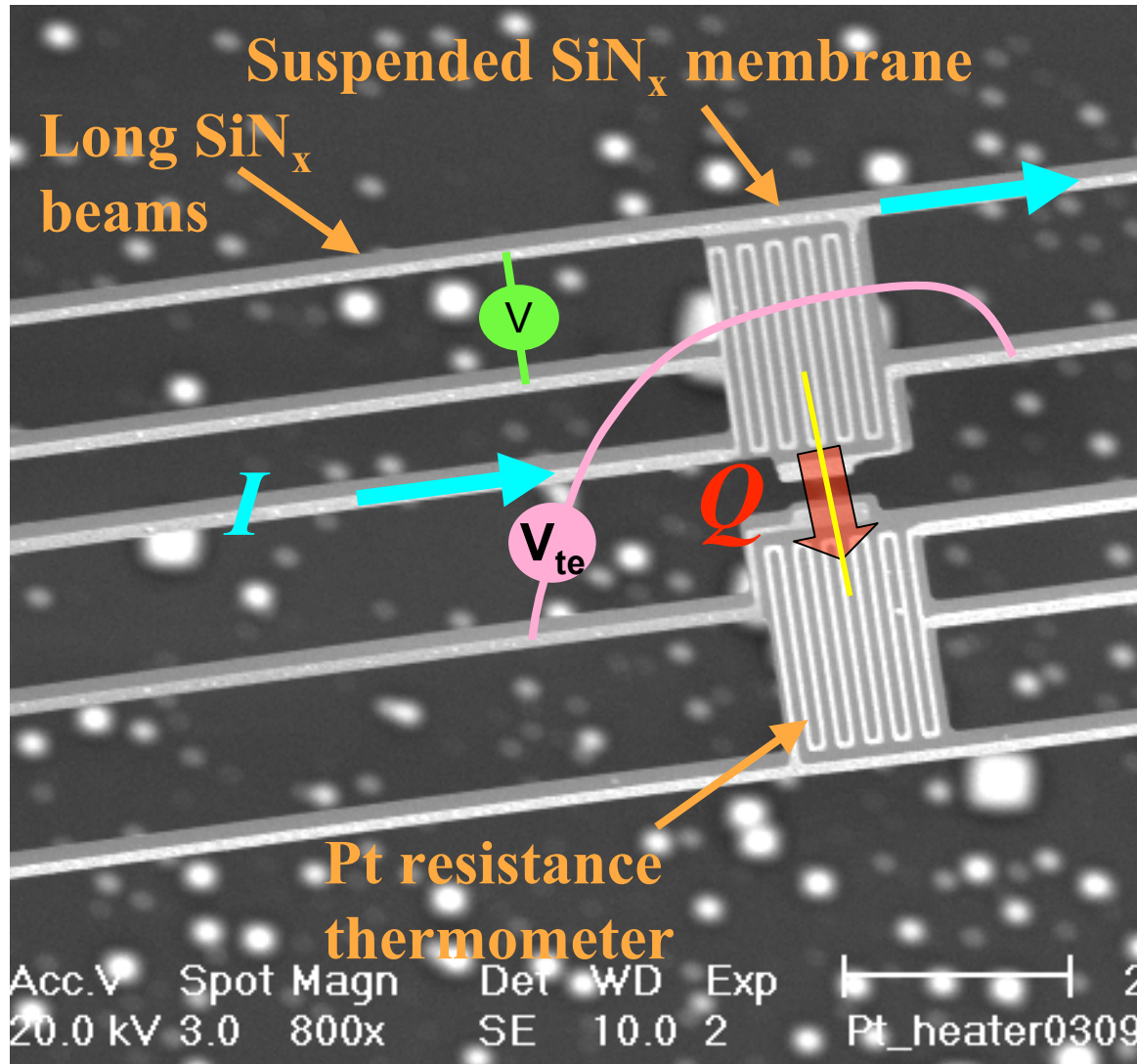
### Information Technology

- Microelectronics
- Data Storage (> 1 TBits/in<sup>2</sup>)
- Wireless (>100 GHz)

### Energy Technology

- Insulation ( $k < 1$  mW/m-K)
- High Cond ( $k > 5000$  W/m-K)
- Thermoelectricity ( $ZT > 3$ )

# Transport in Individual Nanotubes and Nanowires



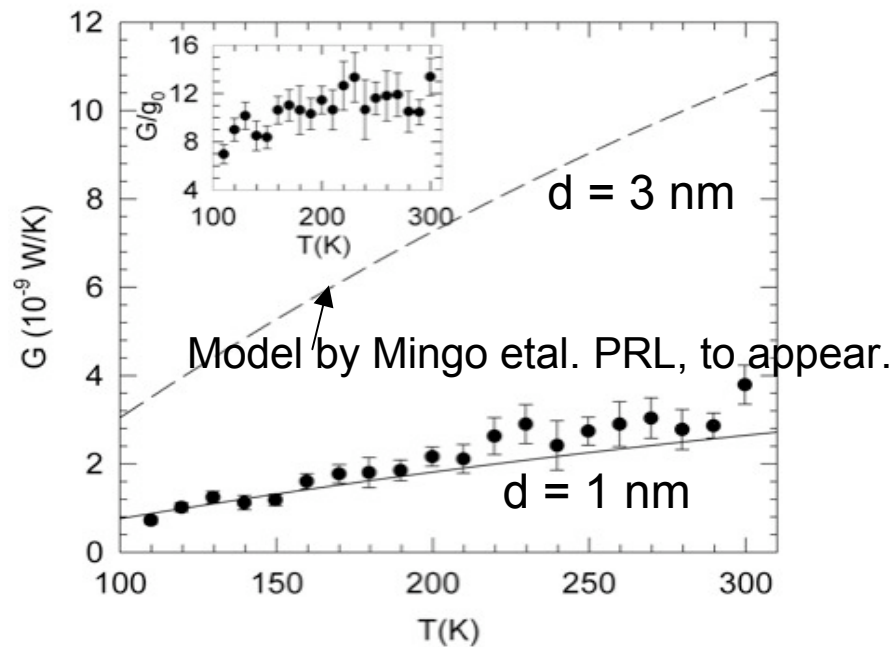
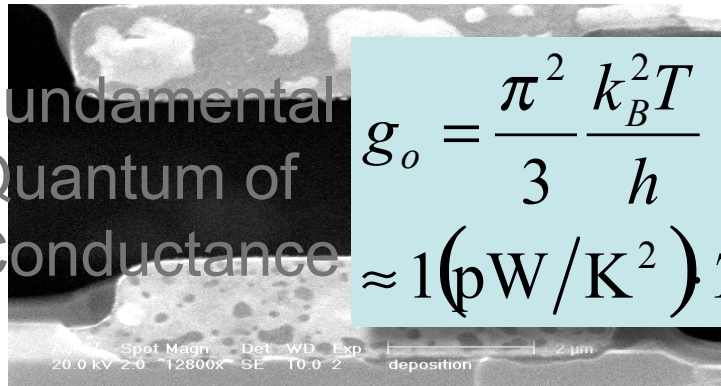
Kim, Shi, Majumdar, McEuen, *PRL* **87**, 215502 (2001); Shi, Li, Yu, Jang, Kim, Yao, Kim, Majumdar, *JHT* **125**, 881 (2003)

# Transport in Individual Nanotubes

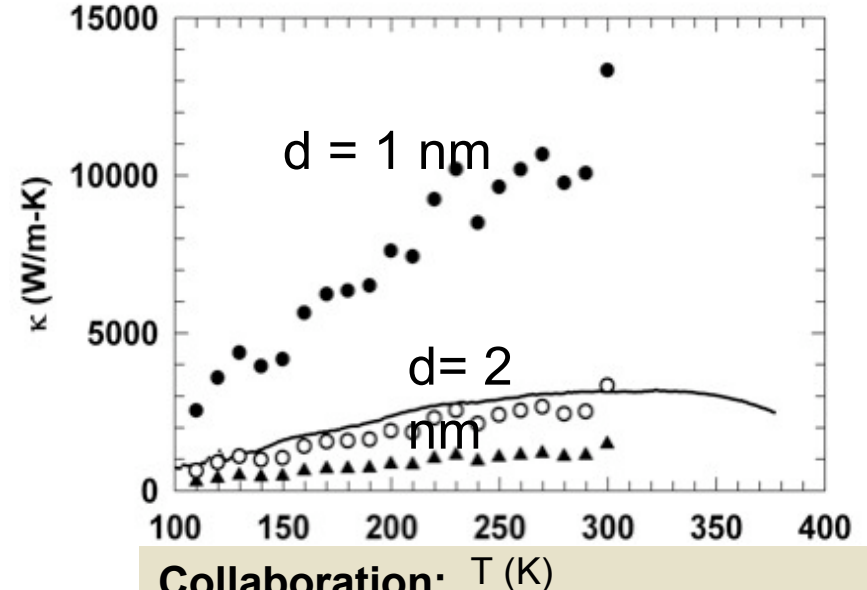
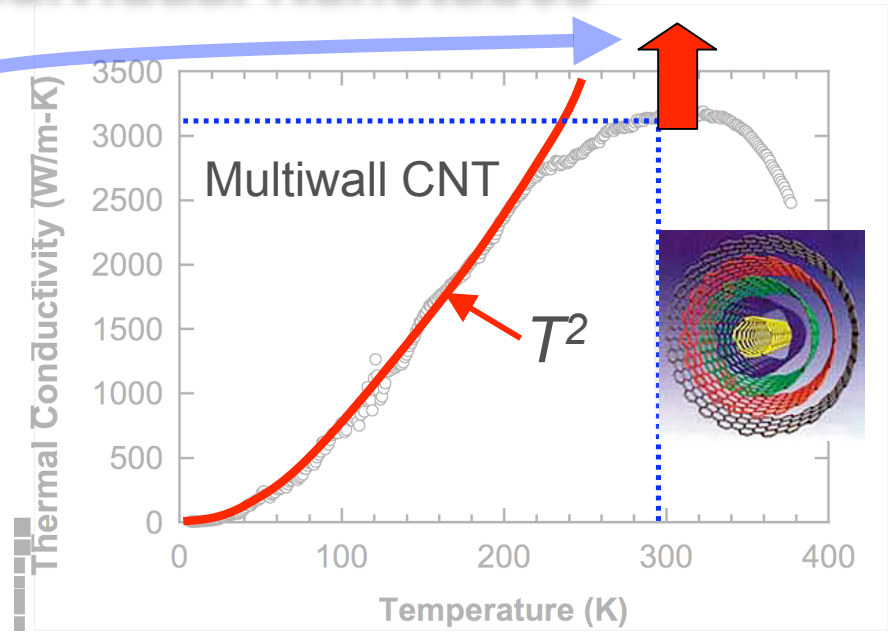
Fundamental  
Quantum of  
Conductance

$$g_o = \frac{\pi^2}{3} \frac{k_B^2 T}{h}$$

$$\approx 1 \left( \text{pW/K}^2 \right) T$$



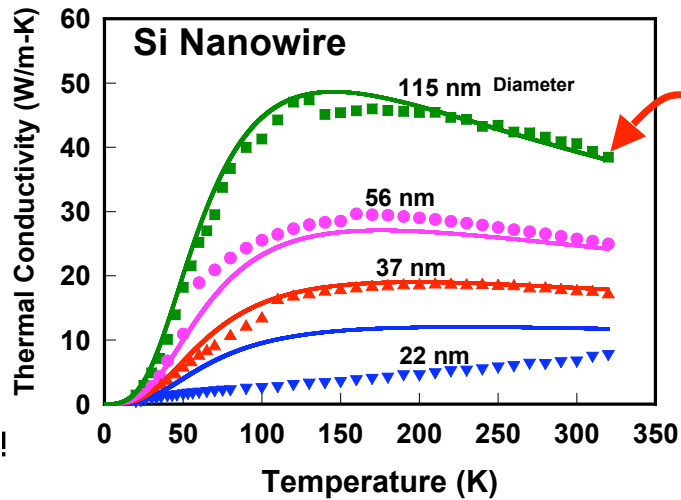
Yu et al., *Nanoletters* (2005)



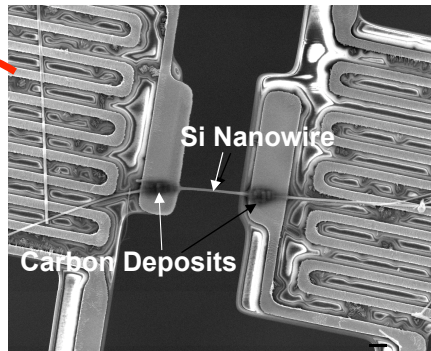
**Collaboration:** T (K)

Li Shi (UT-Austin); P. McEuen (Cornell)  
Philip Kim (Columbia); Alex Zettl (UCB)

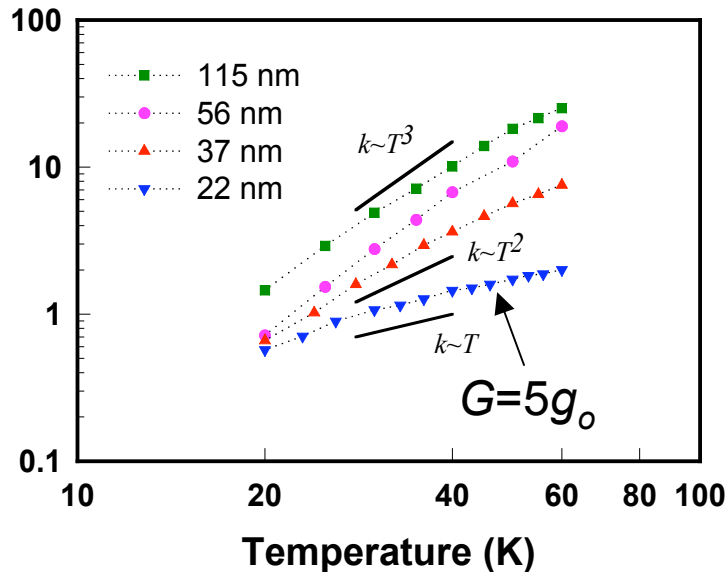
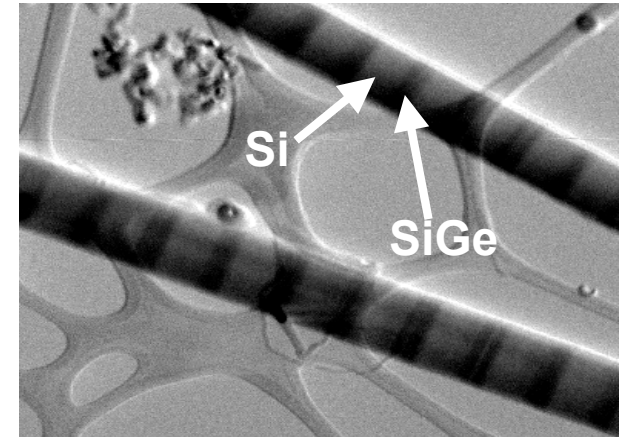
# Transport in Nanowires



Li, ..., Yang, Majumdar, APL (2003a,b)



## SiGe Superlattice NW



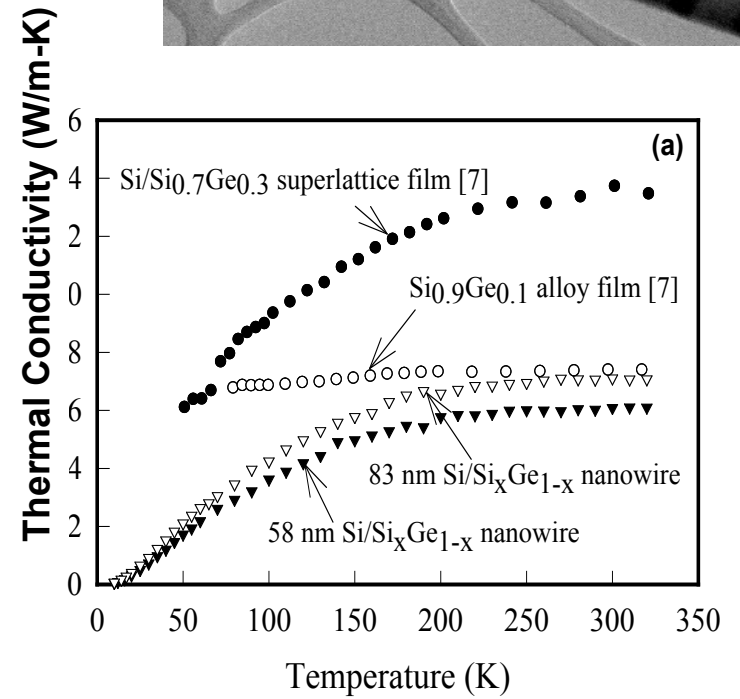
$$g_o = \frac{\pi^2}{3} \frac{k_B^2 T}{h}$$

$$\approx 1 \text{ (pW/K}^2\text{)} T$$

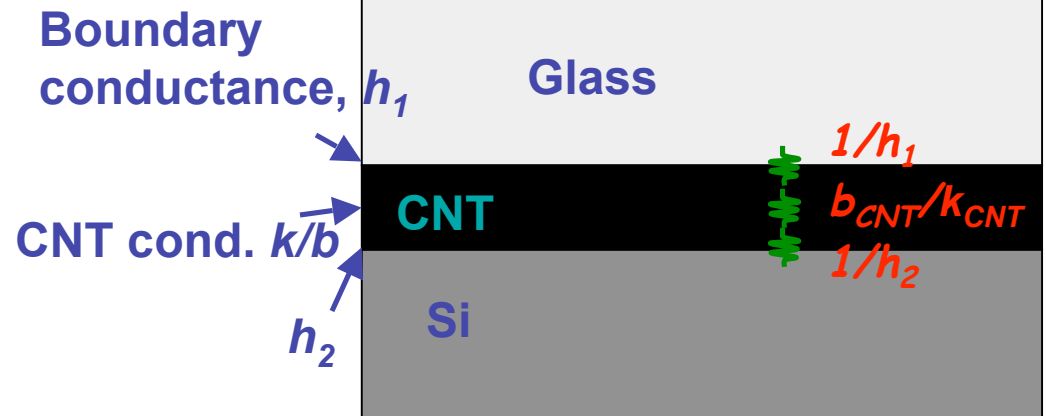
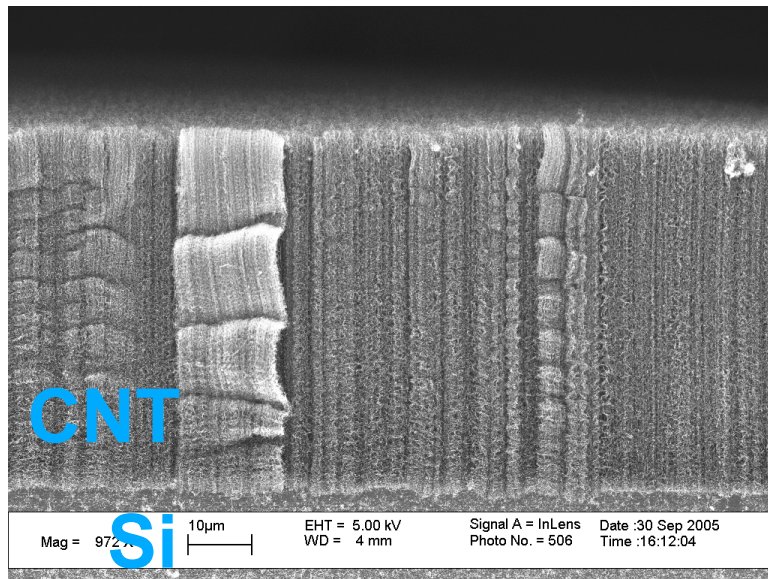
## Collaboration:

Deyu Li (ME, Vanderbilt)

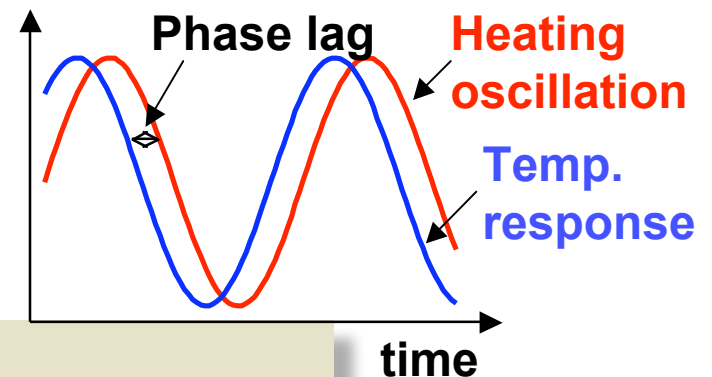
Peidong Yang (Chem, UCB); Joel Moore (Phys, UCB)



# CNT as Thermal Interface Materials



- Harmonically modulated CW laser heating
- Phase detection to determine thermal properties



## CVD MWNT

- Catalyst: 10nm Al / 10nm Fe
- 750 °C, ethylene
- Si or Mo substrate

Tube  $\Phi$  20~30 nm

Height 5 ~ >100  $\mu$ m

Density  $10^{10}$ ~ $10^{11}$  cm<sup>-2</sup>

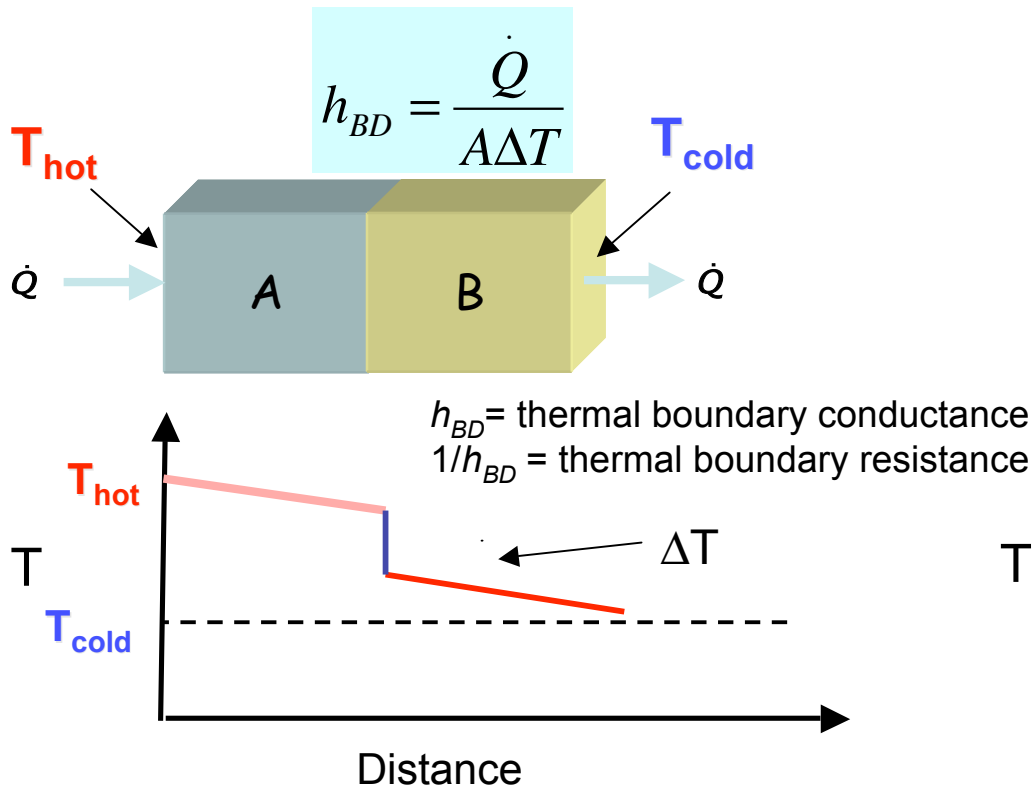
## Collaboration:

Lance Delzeit, M. Meyyappan (NASA-Ames);  
Yang Zhao, Ali Kashani (Atlas Sci.)

## Two Types of Interface Resistance

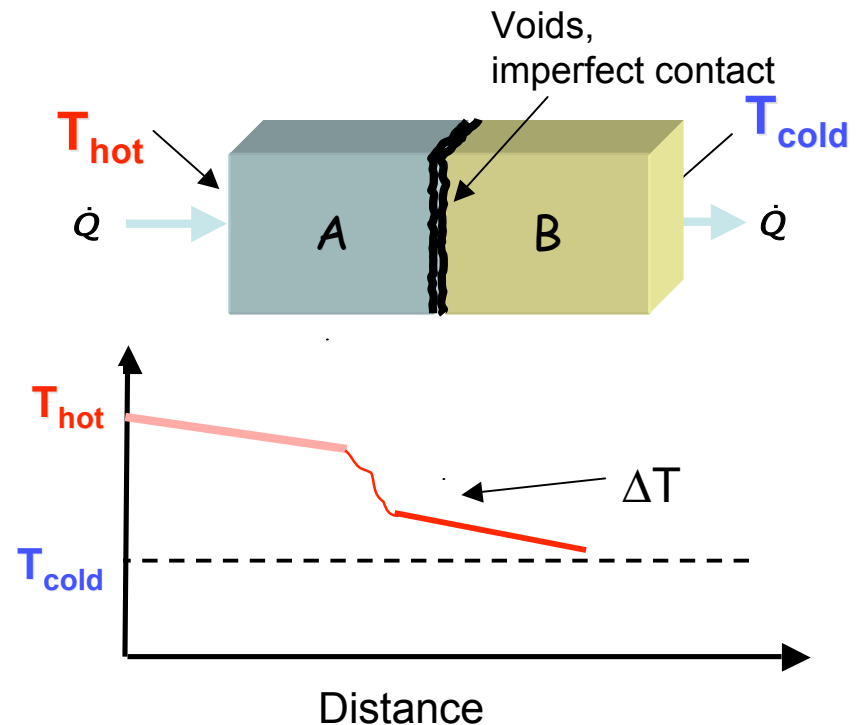
### Thermal Boundary Resistance

- Due to difference in the acoustic properties:  
Phonon reflection at the interface
- Electron-phonon interaction
- Present even in the case of perfect contact with no roughness
- Microscopic quantity

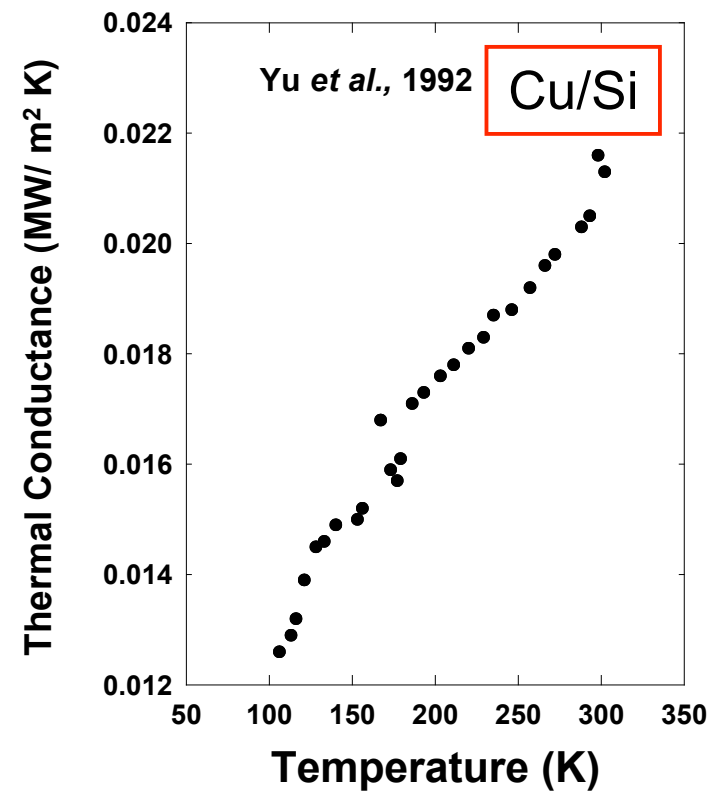
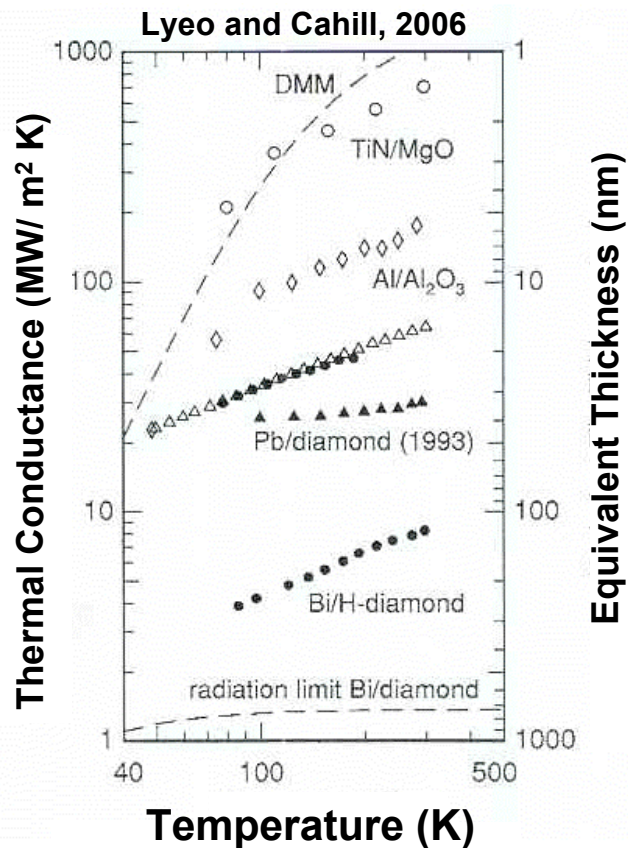
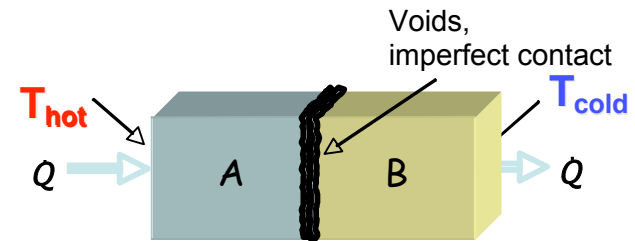
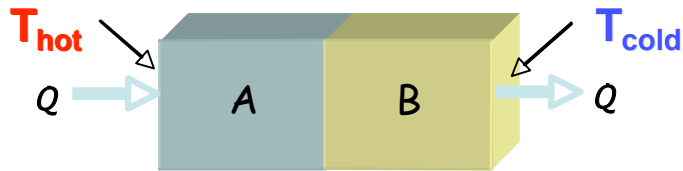


### Thermal Constriction Resistance

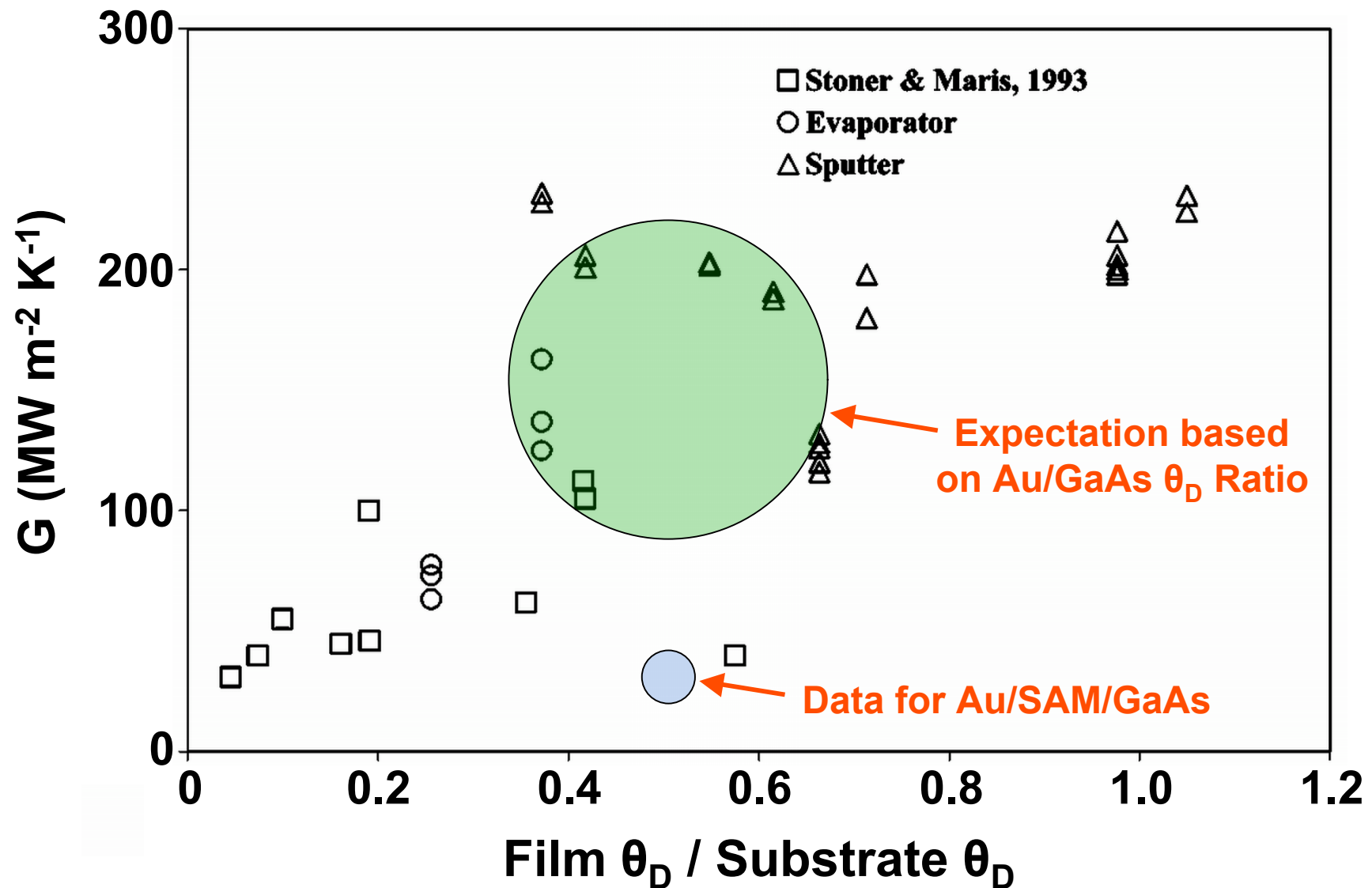
- Important for bulk surfaces
- Macroscopic quantity



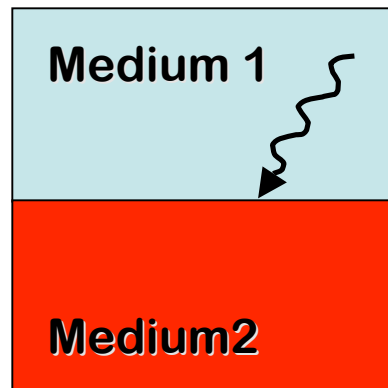
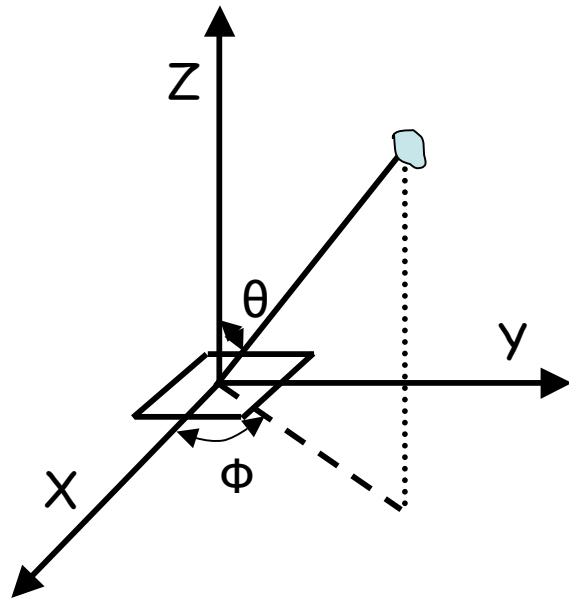
## Both Constriction Resistance and Thermal Boundary Resistance Are Important



# Conductance vs. Debye Temperature Ratio



# Gross Phonon Heat Flux at Interfaces



Thermal Boundary  
Conductance

$$h_{BD} = \frac{q_{1 \rightarrow 2}(T_1) - q_{2 \rightarrow 1}(T_2)}{(T_1 - T_2)}$$

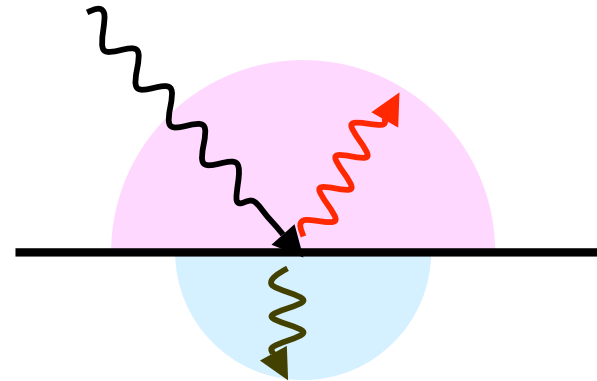
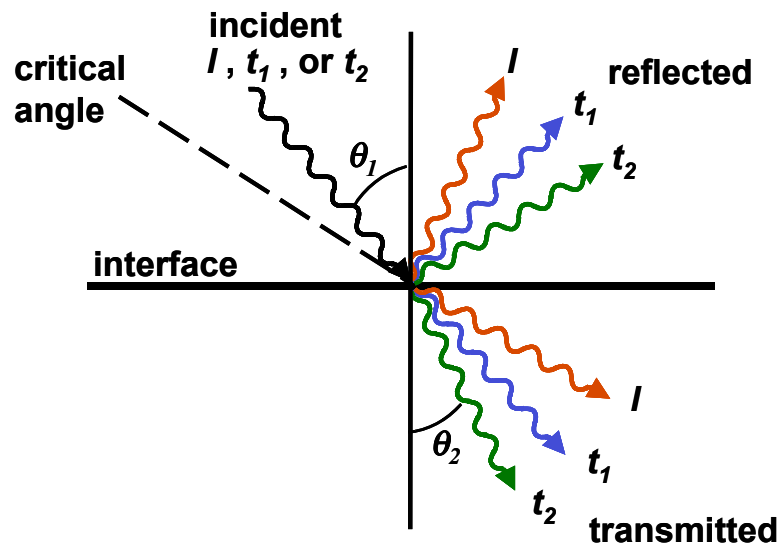
$$q_{1 \rightarrow 2} = \sum_j \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^{\omega_{\max}} \underbrace{\alpha_{1 \rightarrow 2}(\theta, j, \omega)}_{\text{Transmission Probability}} \underbrace{N_{1,j}(\omega, T) \hbar \omega}_{\text{Energy Distribution}} \underbrace{\frac{1}{4\pi} v_{g,1,j}(\omega) \cos \theta \sin \theta d\theta d\phi d\omega}_{\text{Geometric Considerations}}$$

Transmission  
Probability

Energy  
Distribution

Geometric Considerations

# Transmission Probabilities



## Acoustic Mismatch Model (AMM)

$$\alpha_{1 \rightarrow 2} = \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2}$$

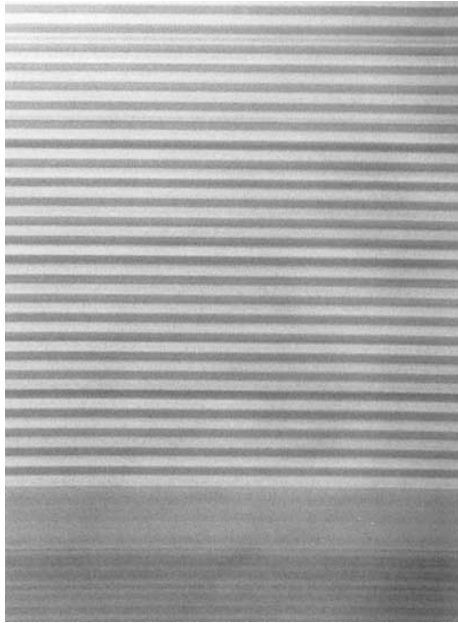
$$Z_i = \rho_i c_i$$

## Diffuse Mismatch Model (DMM)

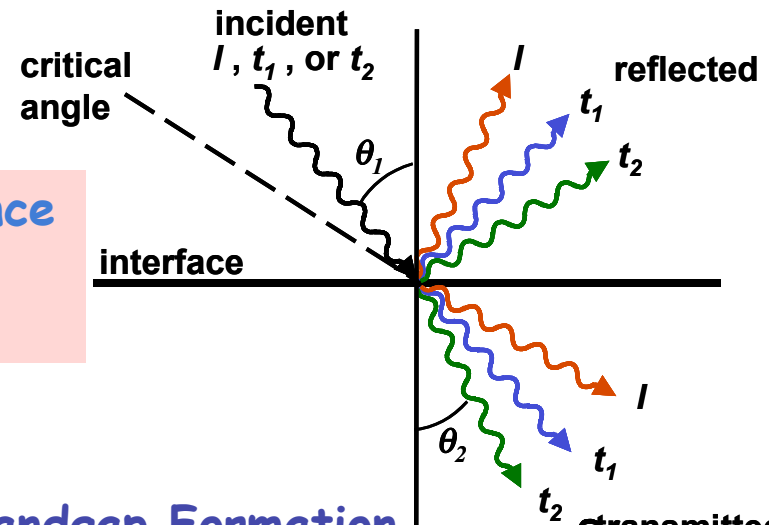
$$\alpha_{1 \rightarrow 2}(\omega) = \frac{[\sum_j N_{2,j}(\omega) c_{2,j}(\omega)]}{[\sum_{i,j} N_{i,j}(\omega) c_{i,j}(\omega)]}$$

# Thermal Transport in Semiconductor Superlattices

TEM



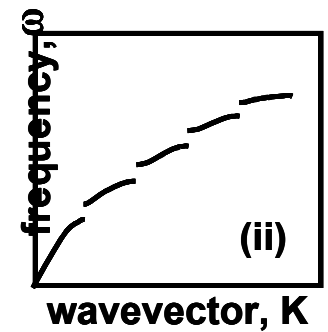
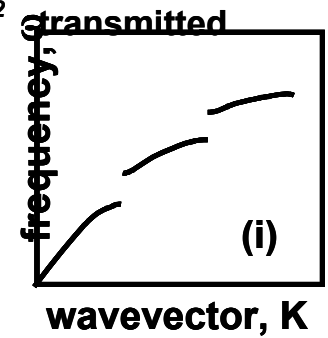
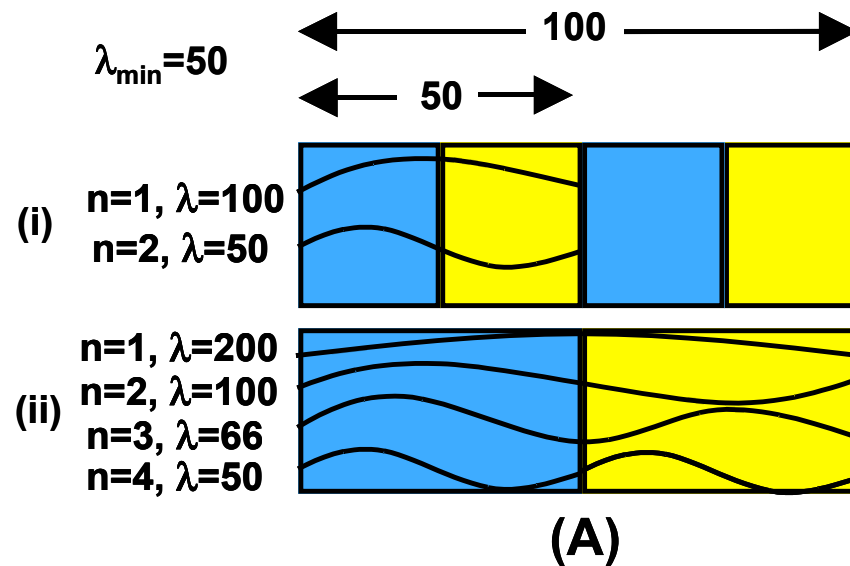
## Phonon Reflection/Transmission



Acoustic Impedance Mismatch (AIM)  
 $= (\rho v)_1 / (\rho v)_2$

## Phonon Bandgap Formation

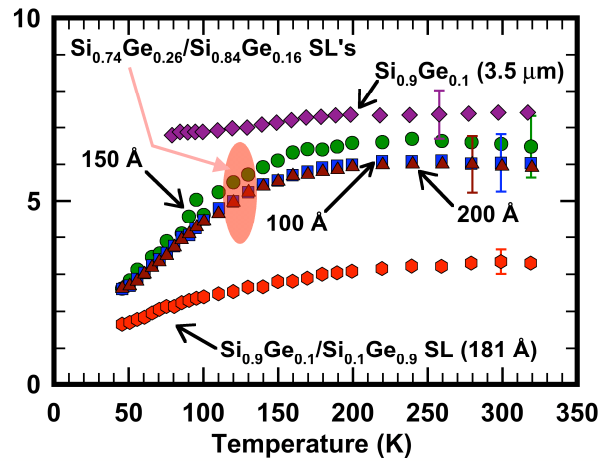
$$n\lambda = 2d \sin\theta$$



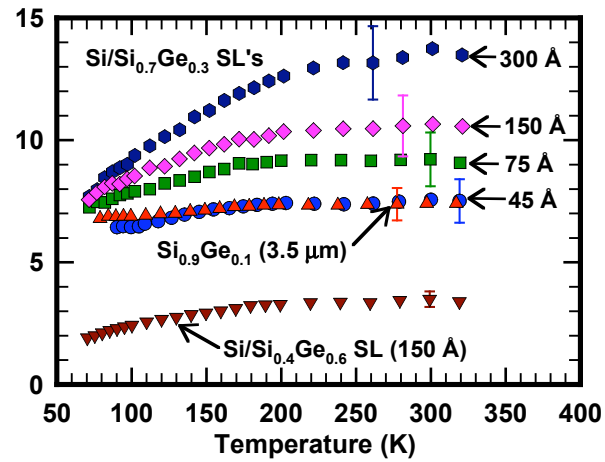
(B)

# Si-Ge Superlattices

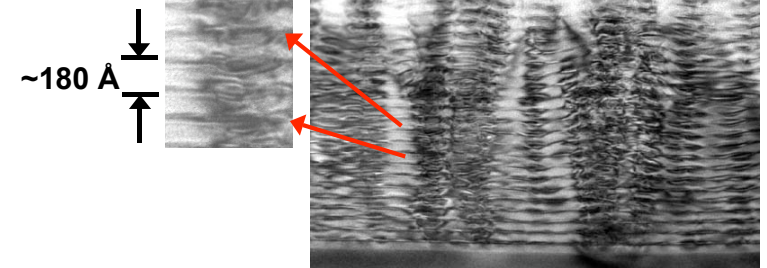
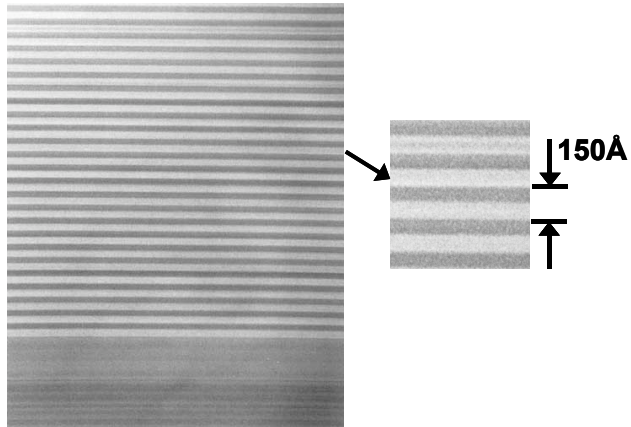
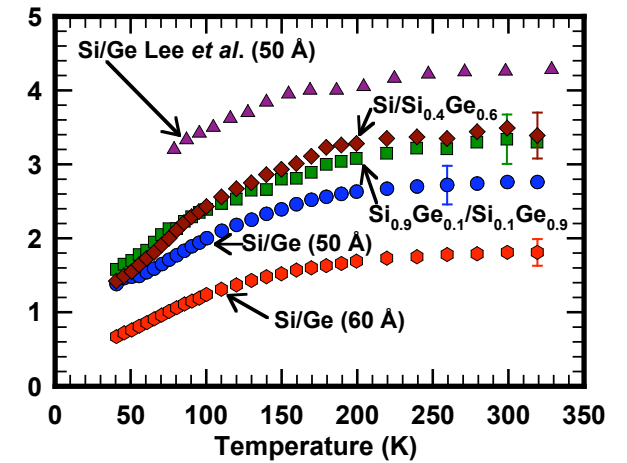
$\text{Si}_x\text{Ge}_{1-x}/\text{Si}_y\text{Ge}_{1-y}$ :  $|x-y| \leq 0.1$



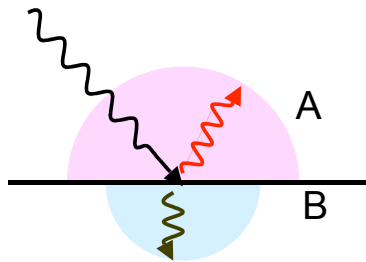
$\text{Si}_x\text{Ge}_{1-x}/\text{Si}_y\text{Ge}_{1-y}$ :  $|x-y| = 0.3$



$\text{Si}_x\text{Ge}_{1-x}/\text{Si}_y\text{Ge}_{1-y}$ :  $|x-y| \geq 0.6$



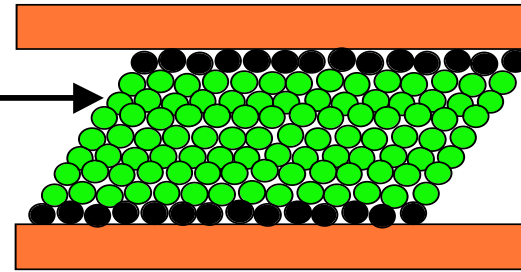
# Molecular Heterostructures



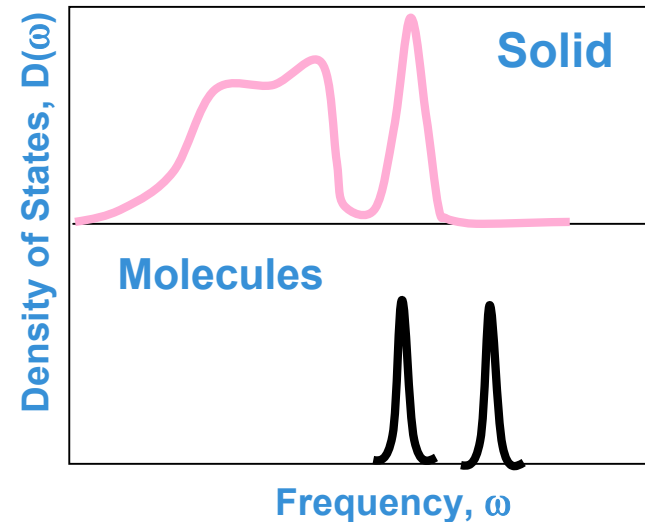
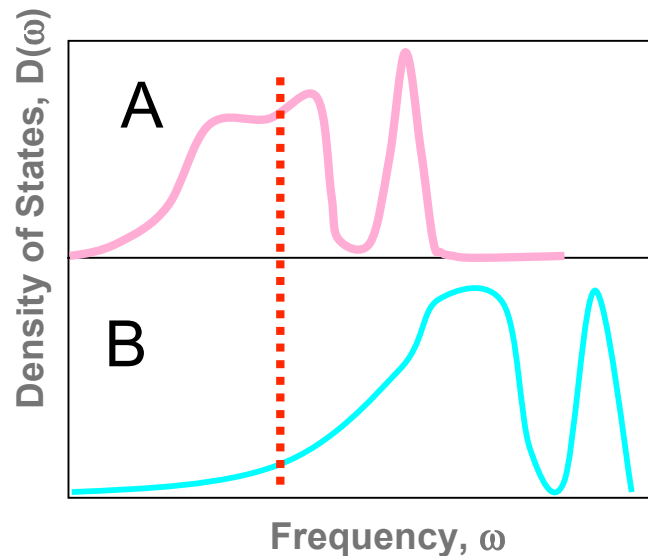
**Diffuse Mismatch Model (DMM)**  
Swartz and Pohl (1989)

Alkane Dithiols  
N-Benzene Dithiols

## Molecular Heterostructures



**10 MW/m<sup>2</sup>-K**   **Conductance**   **1 GW/m<sup>2</sup>-K**  
@300K



### Collaboration:

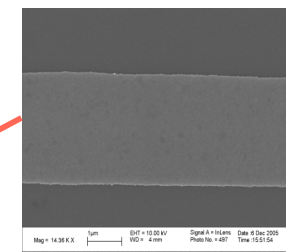
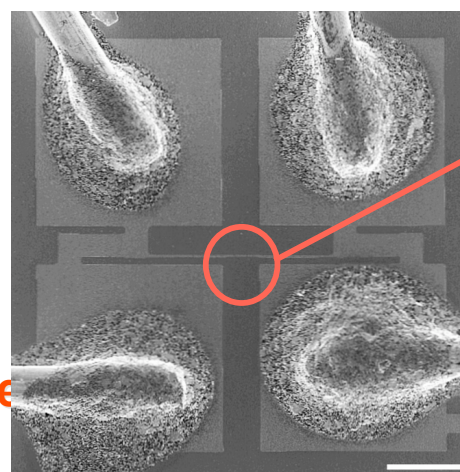
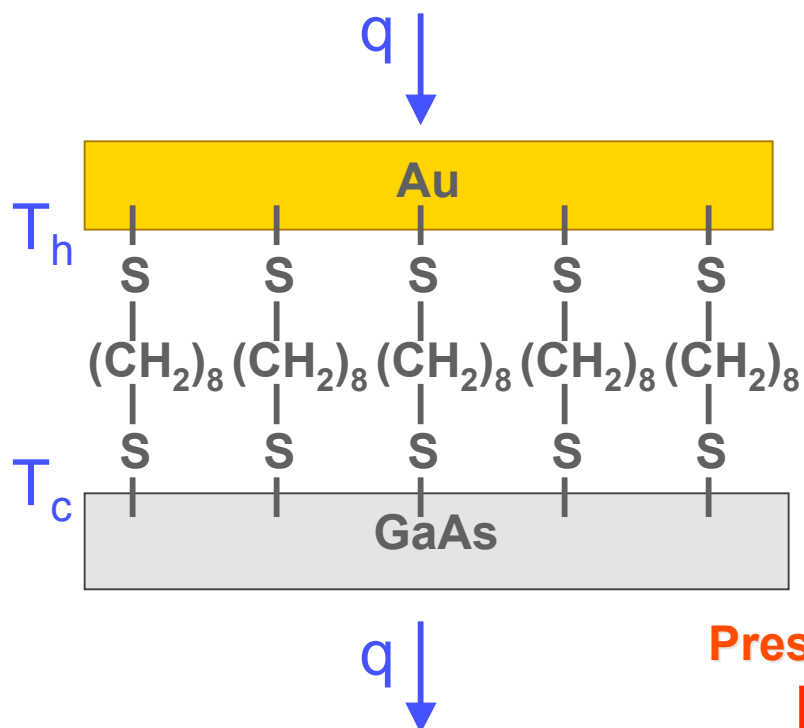
Rachel Segalman (ChemE, UCB) - Synthesis  
Pawel Keblinski (Matl Sci-RPI) - Theory  
Jack Lloyd (ME, MSU) - Theory

# Molecular Interfaces

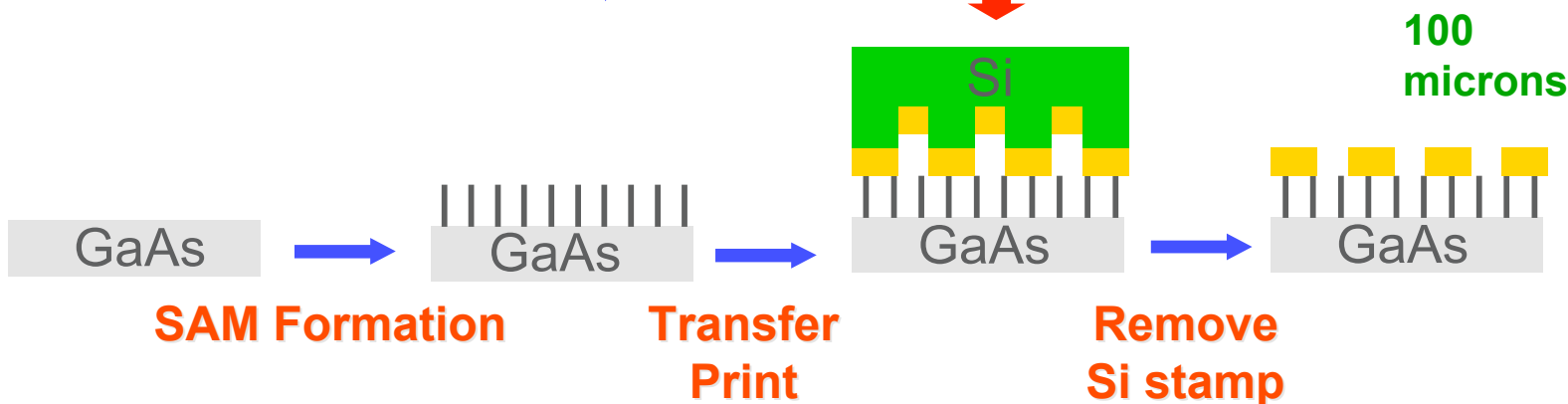


Robert Wang

$$G = \frac{q}{\Delta T}$$

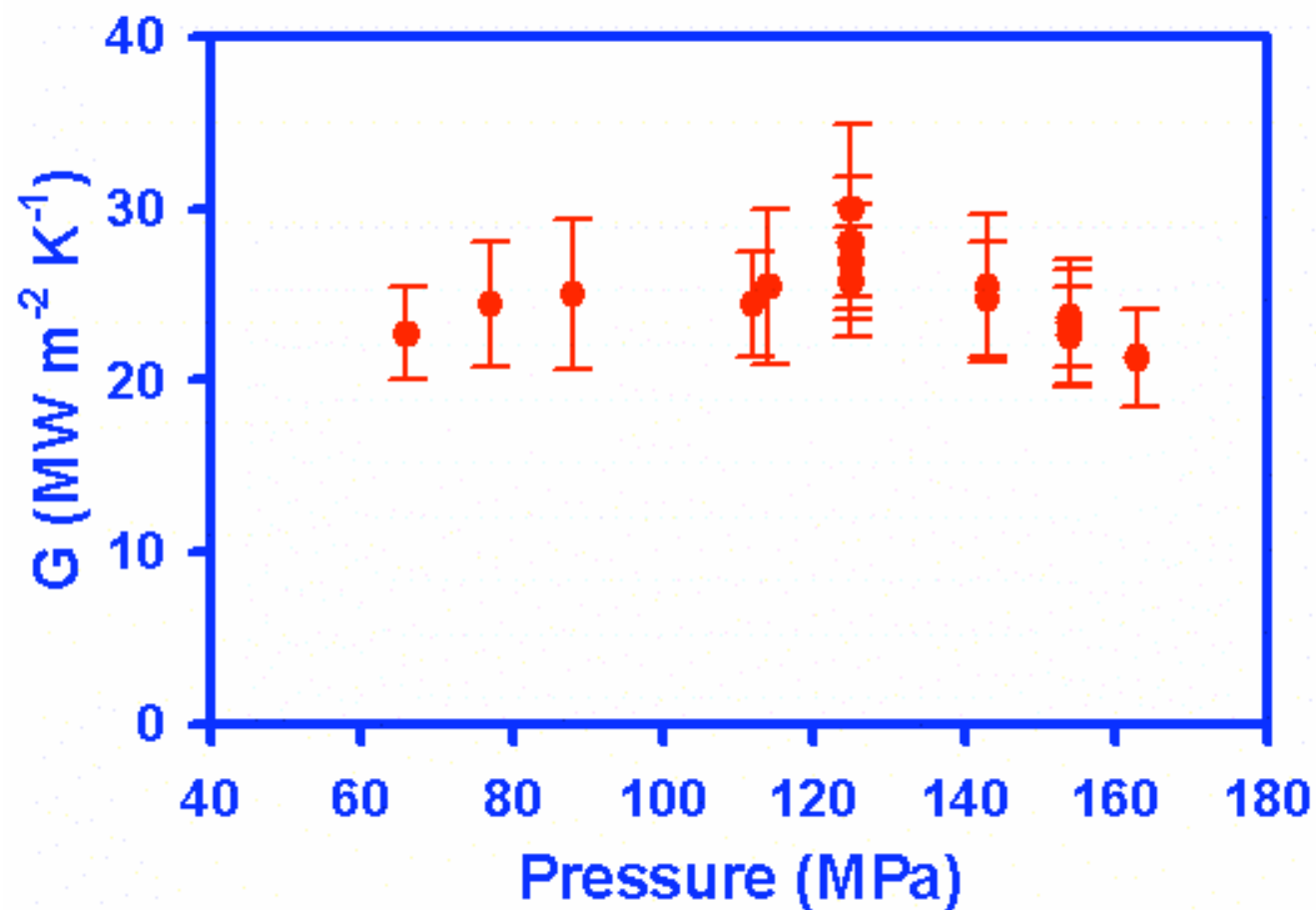


Pressure



Loo et al., *J. Vac. Sci. Technol. B* **20**, 2853 (2002)

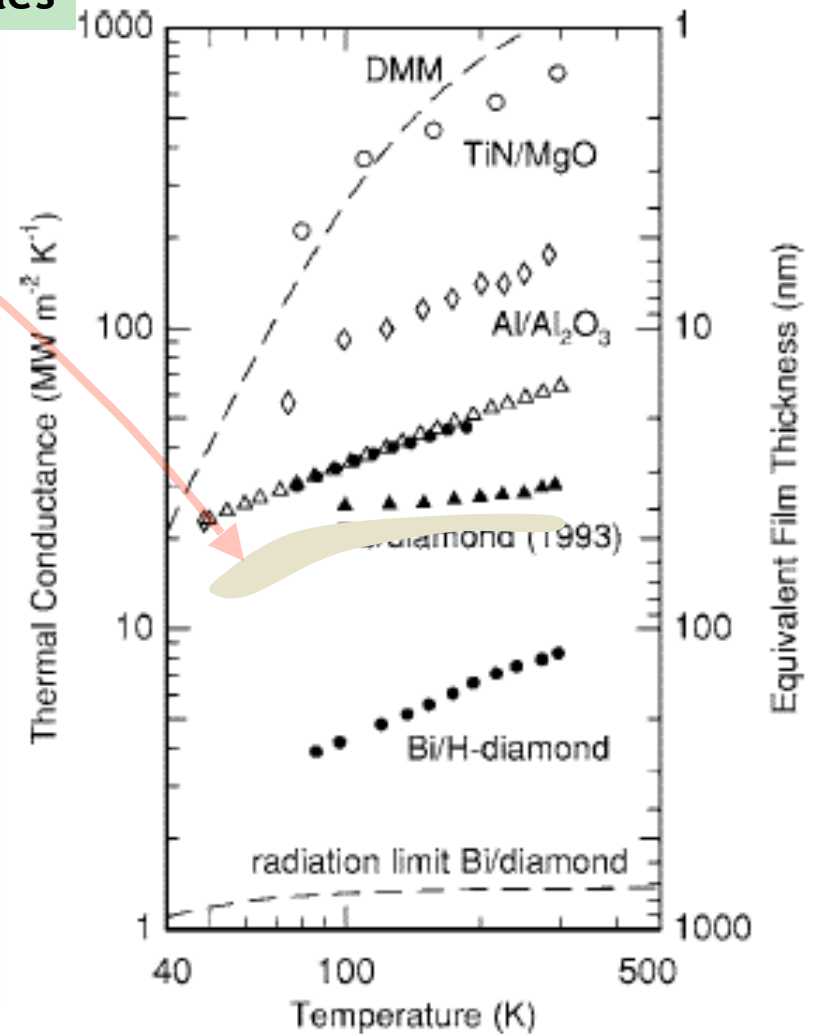
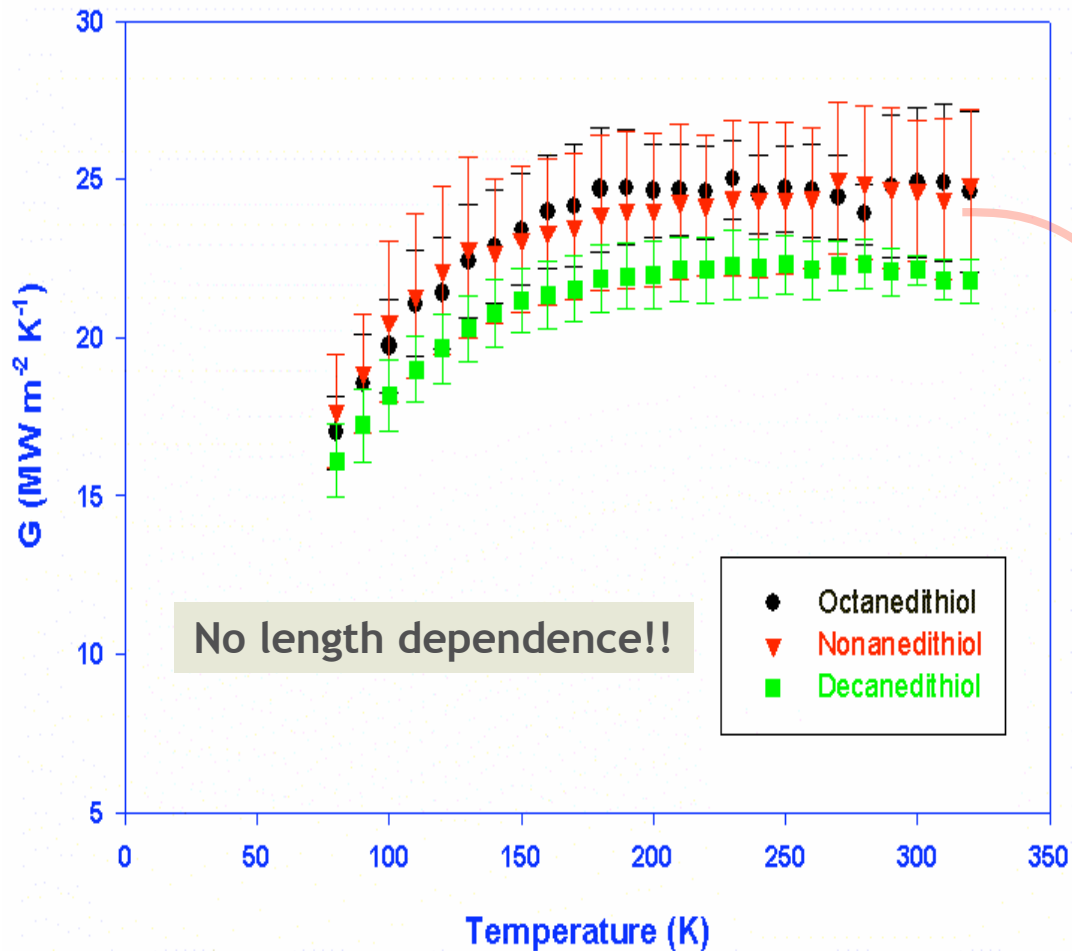
## Effect of Fabrication Pressure



Wang, Segalman, Majumdar, *Appl. Phys. Lett.* (2006)

# Thermal Conductance of Molecular Interfaces

Next: Thermal conductance of aromatic molecules

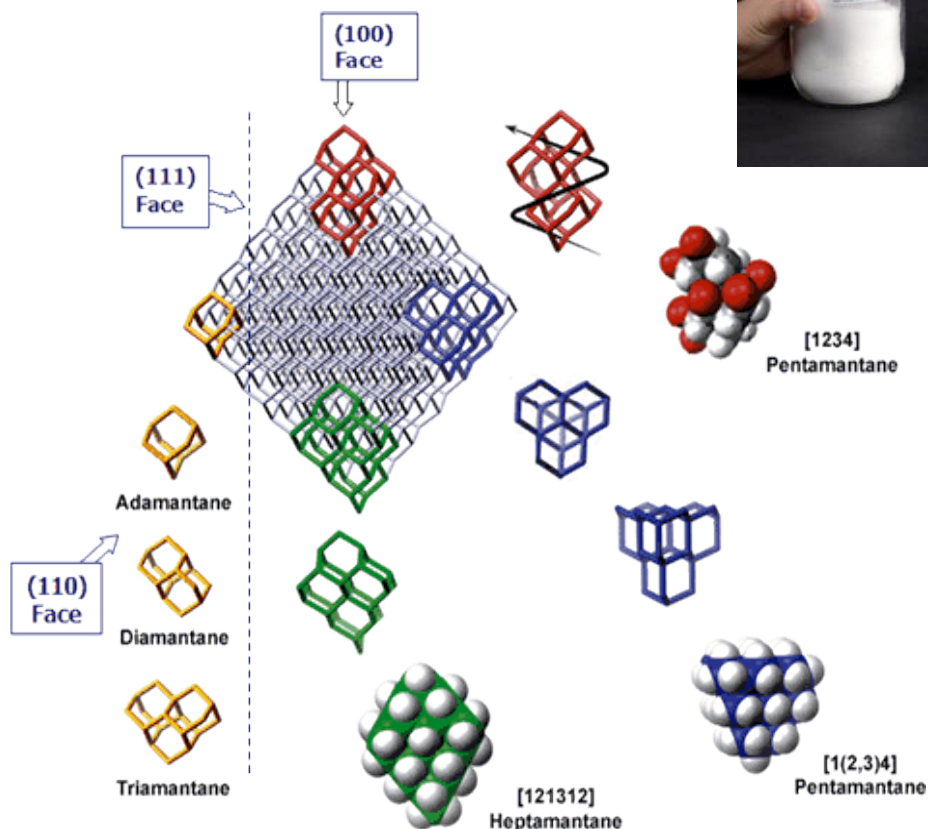


Lyeo, Cahill, Phys. Rev. B. 74, 144301 (2006)

# Molecular Heterostructures

## Diamondoids

$sp^3$  bonded carbon nanostructures



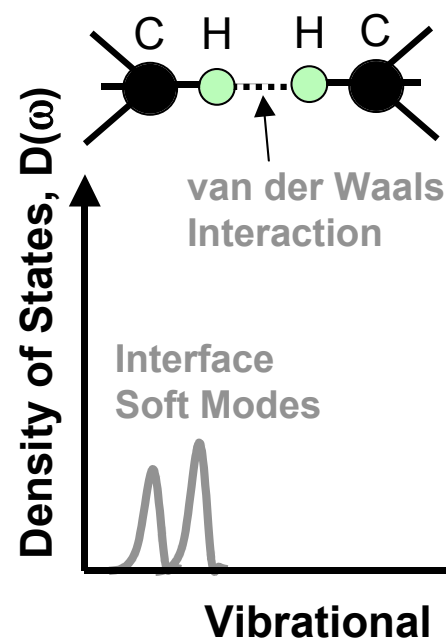
Chevron



## Possible Values

$$G \sim 1 \text{ MW/m}^2\text{-K}$$

$$k \sim 1\text{-}10 \text{ mW/m-K}$$



## Collaboration:

Jeremy Dahl (C-T) - Synthesis  
Pawel Keblinski (RPI) - Theory

**Phonon Optics at Low Temperature (1970s--.)**  
(Narayanamurti, Dynes, et al;  
Wolfe et al.)

**Quantum Phonon Conductance**  
(Schwab, Roukes, 2000)

| Molecular Heterostructures |                                   | Nanowires & Arrays                  |  | Nanotubes                         |
|----------------------------|-----------------------------------|-------------------------------------|--|-----------------------------------|
| Phonon Filtering           | Correlated scattering & diffusion | Uncorrelated scattering & diffusion |  | Waveguiding & ballistic transport |

Ultralow

Polymers

Diamond

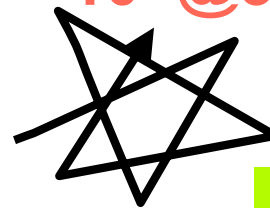
Ultrahigh

**Phonon Length Scales (300K)**

Wavelength = 1-10 nm

Mean Free Paths = 10-100 nm

**$10^4$  @300K**



**Thermal Conductivity**

$$k = Cvl/3$$

### Information Technology

- Microelectronics
- Data Storage (> 1 TBits/in<sup>2</sup>)
- Wireless (>100 GHz)

### Energy Technology

- Insulation ( $k < 1$  mW/m-K)
- High Cond ( $k > 5000$  W/m-K)
- Thermoelectricity ( $ZT > 3$ )