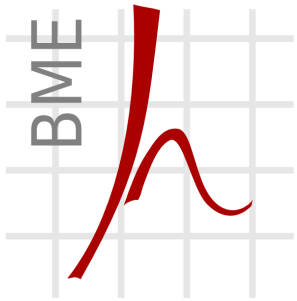
Binary trees

Basics of Programming 1



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2 December, 2020

Content

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

Chapter 1

Binary trees

Trees

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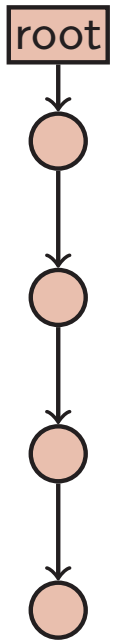
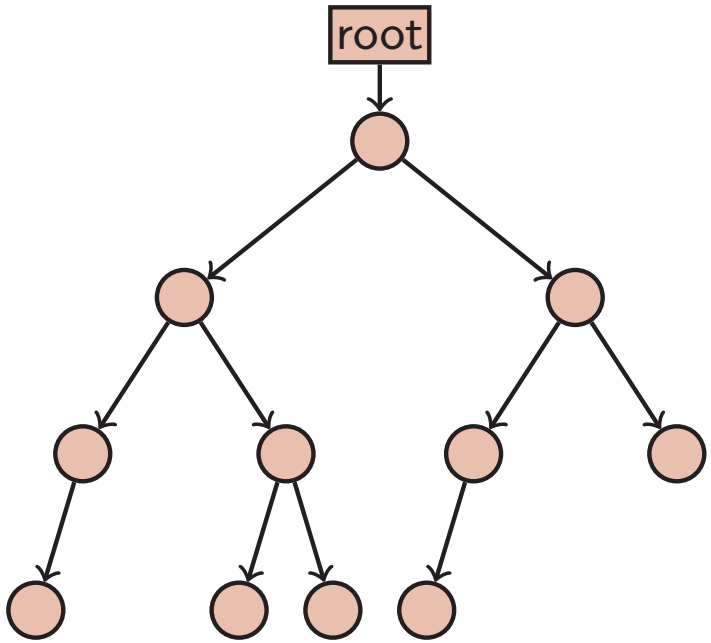


$K = 1$ (linked list)



$K = 2$ (binary tree)

- An acyclic graph
- Every node has exactly one incoming edge
- K-ary tree: every node has at most K outgoing edges

Binary trees

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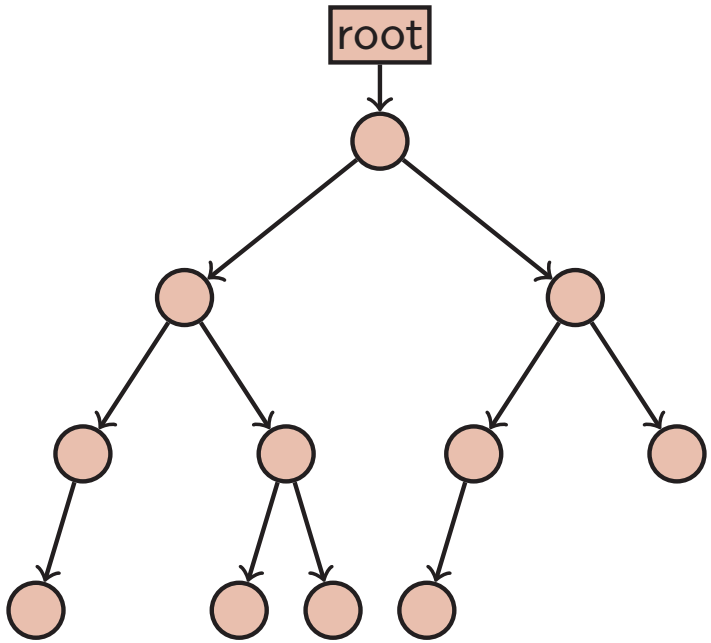

- Declaration of the binary tree data structure

```
1  typedef struct tree {
2    int data;
3    struct tree *left, *right;
4  } tree_elem, *tree_ptr;
```

link

- Typically we typedef not only the struct, but also the pointer

Binary search trees

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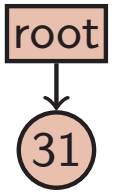

- Sub-tree to the left: only elements smaller than the node
- Sub-tree to the right: only elements greater than the node
- The structure of the tree depends on the insertion order of the elements!

Binary search trees

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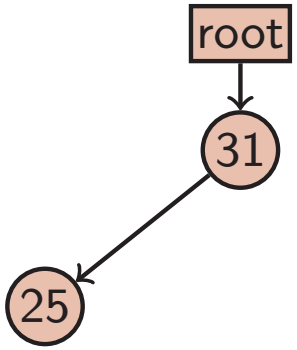

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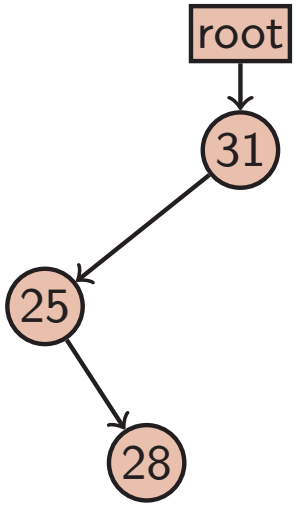

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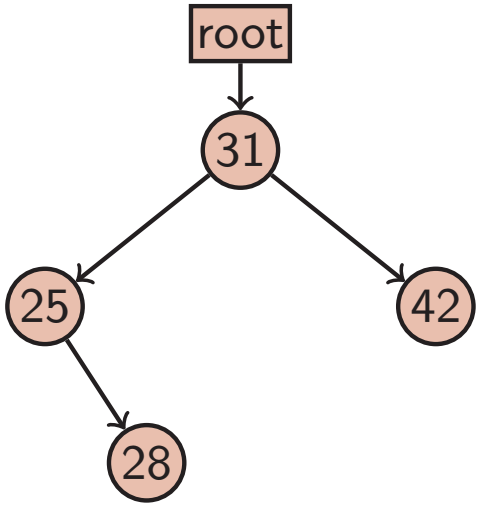

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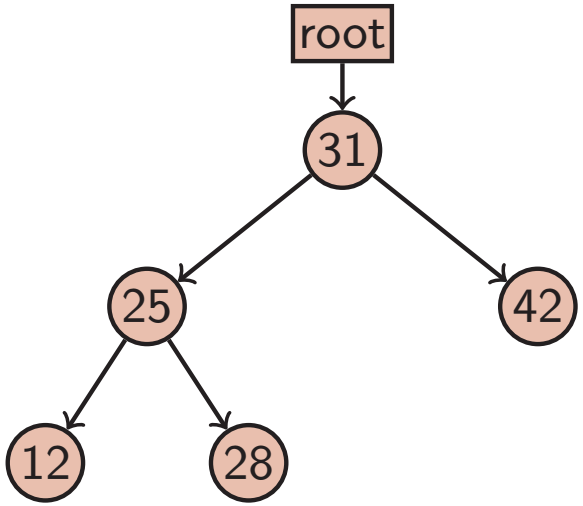

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Binary search trees

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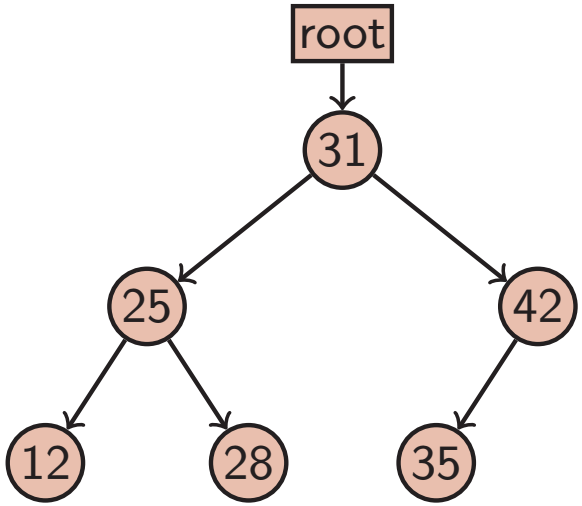

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Binary search trees

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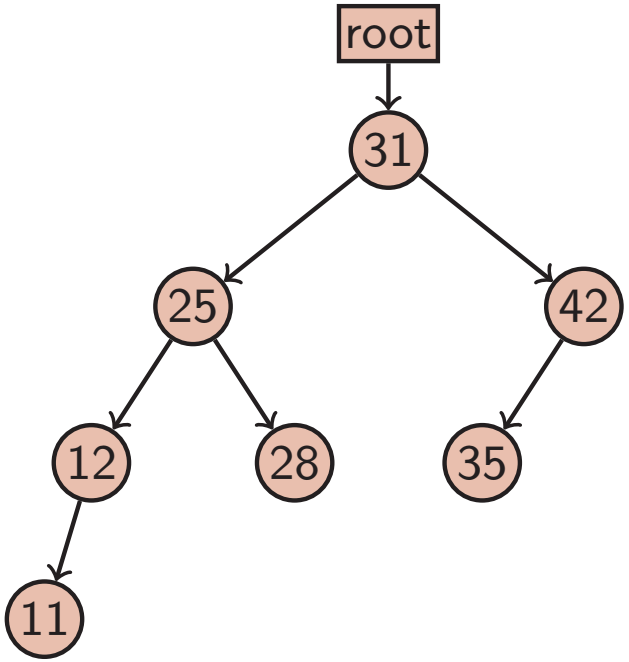

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Binary search trees

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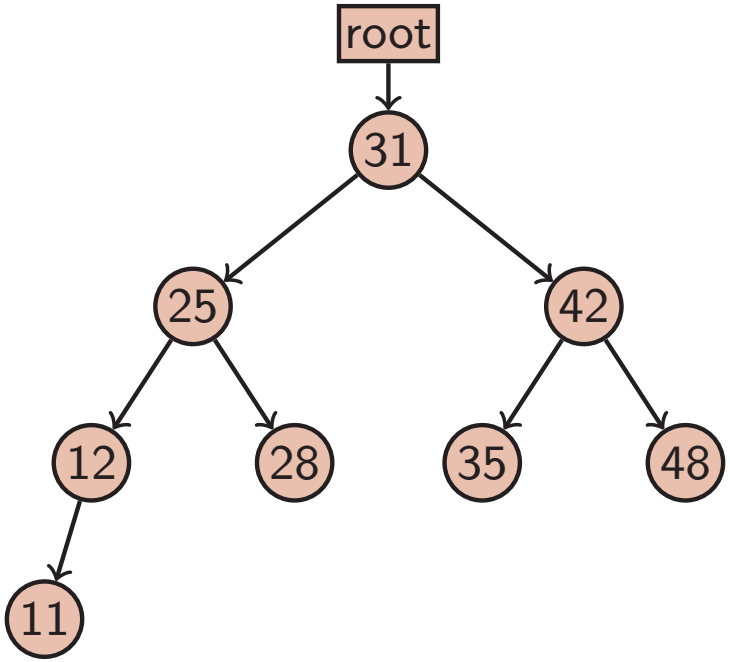

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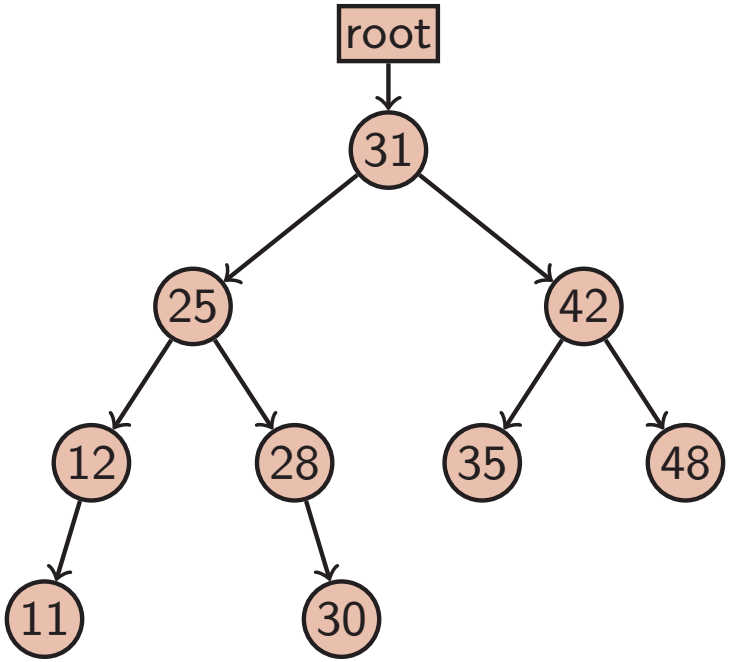

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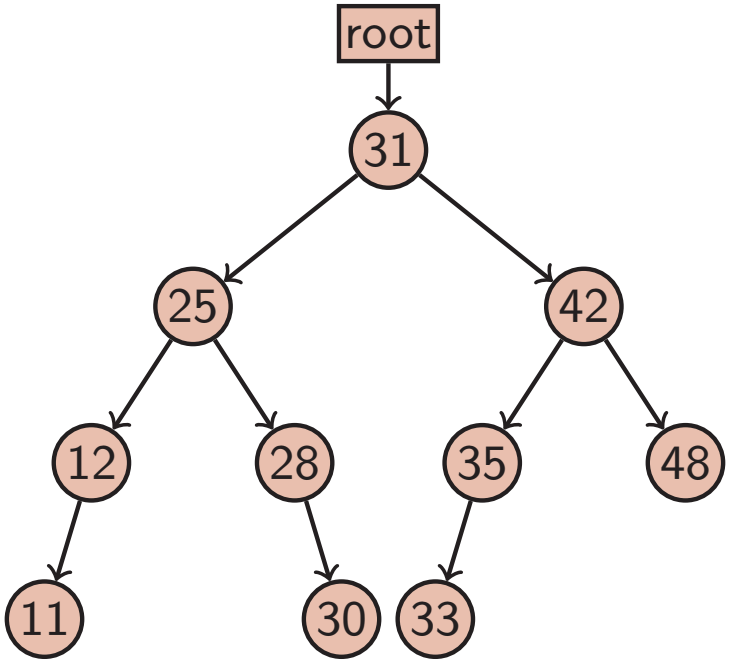

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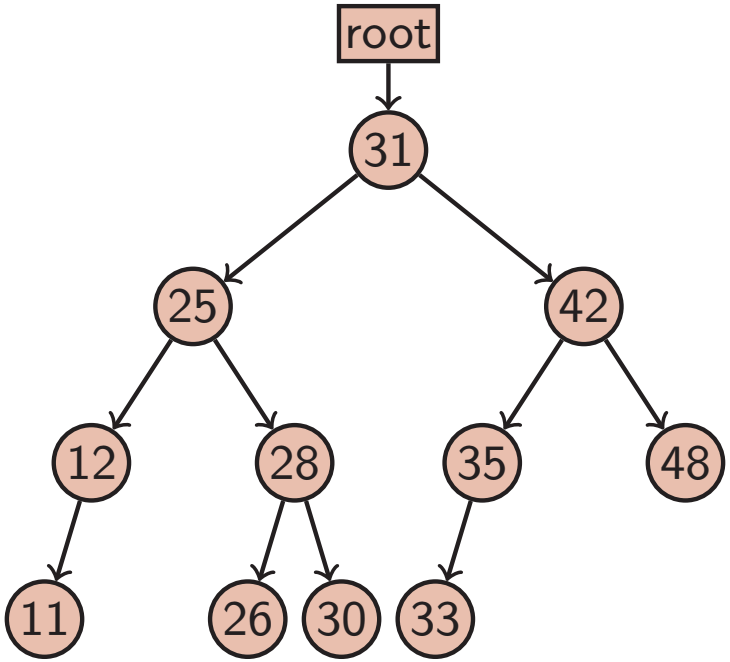

31 25 28 42 12 35 11 48 30 33 26

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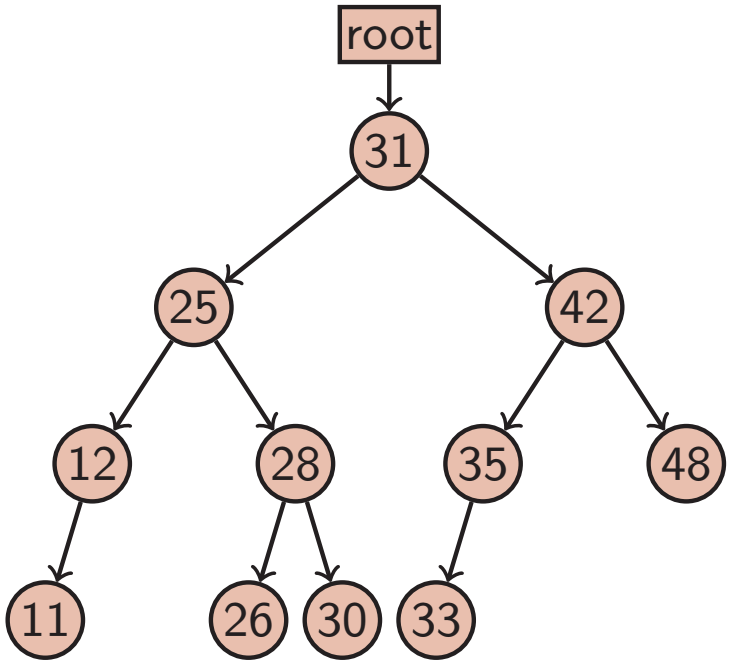
Searching an element in the tree




```
1  tree_ptr find(tree_ptr root,
2                int data)
3  {
4    while (root != NULL &&
5      data != root->data)
6    {
7      if (data < root->data)
8        root = root->left;
9      else
10       root = root->right;
11   }
12   return root;
13 }
```

link

Searching an element in the tree

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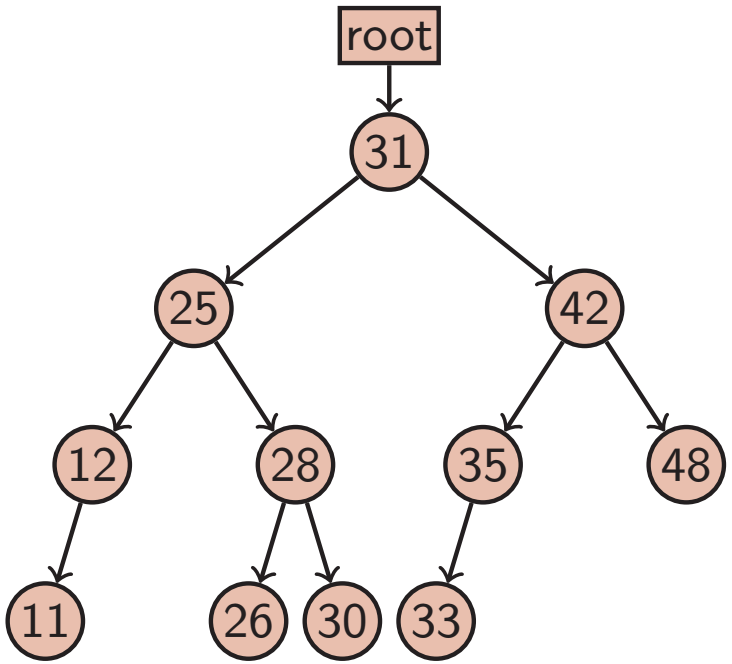


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link

- This is not recursive yet

Searching an element in the tree

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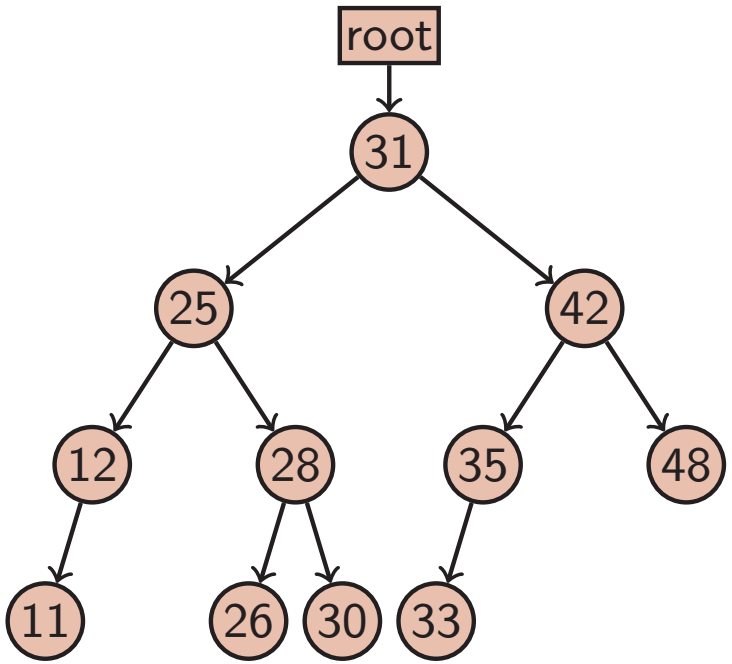


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link

- This is not recursive yet
- In a depth-d tree the max. number of steps is d

Searching an element in the tree

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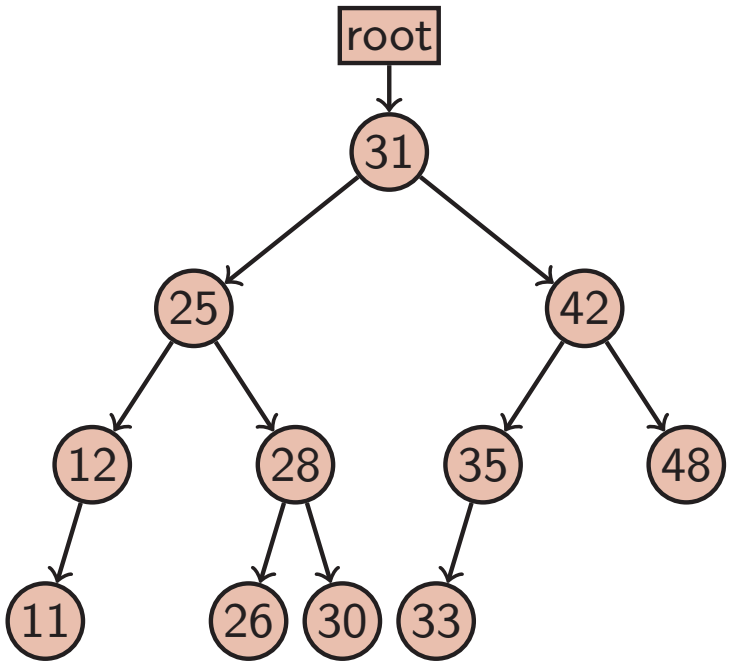


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11   }
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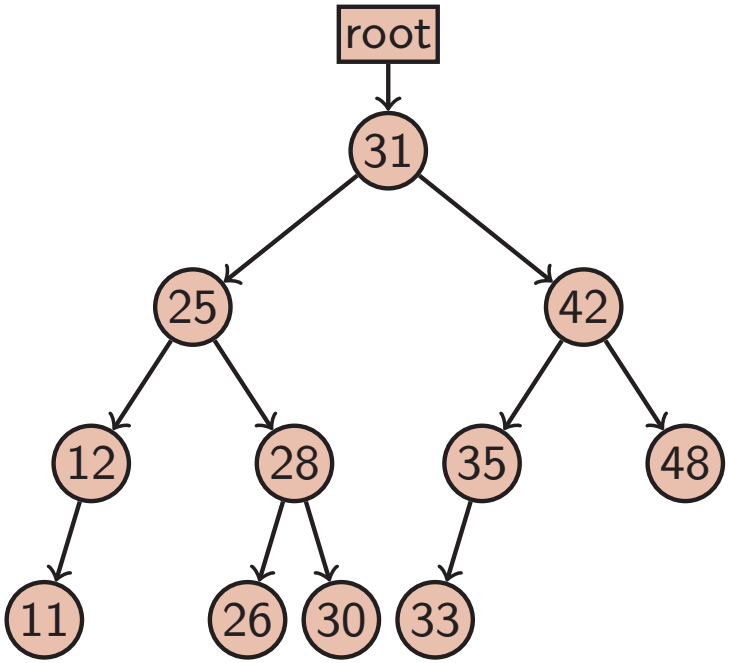
link

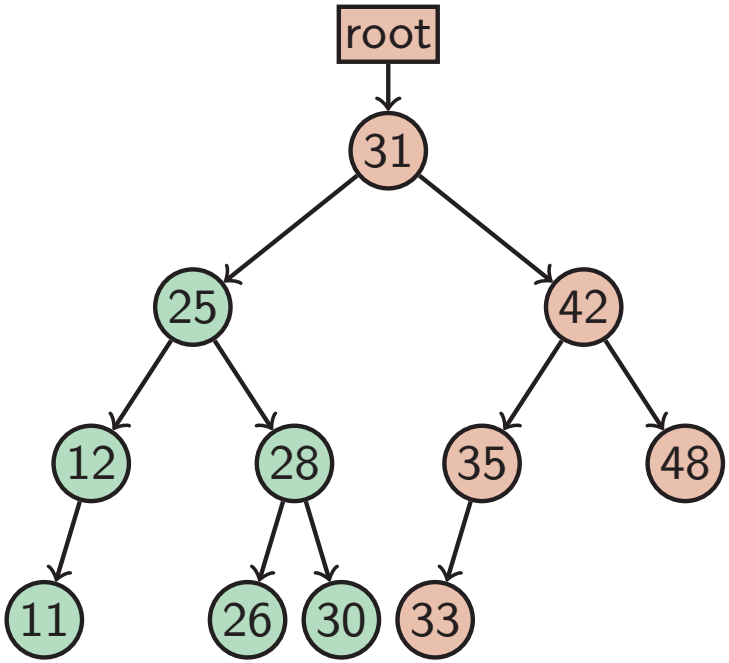
- This is not recursive yet
- In a depth-d tree the max. number of steps is d
- If the tree is balanced and has n elements $\Rightarrow \approx \log_2 n$ steps!

In-order traversal




```
1  void inorder(tree_ptr root)
2  {
3    if (root == NULL)
4      return;
5    inorder(root->left);
6    printf("%d ", root->data);
7    inorder(root->right);
8  }
```

In-order traversal

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5    inorder(root->left);
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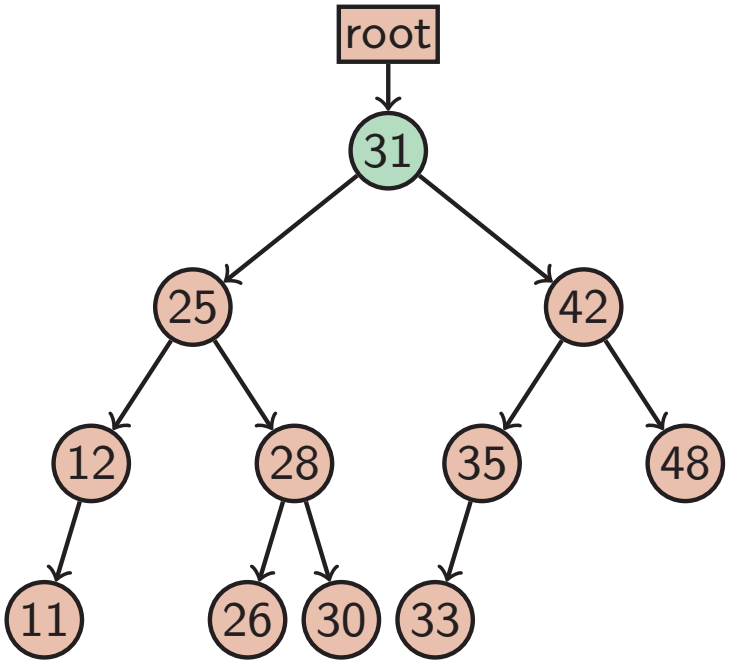
7    inorder(root->right);
8  }
```

```
11 12 25 26 28 30
```

In-order traversal

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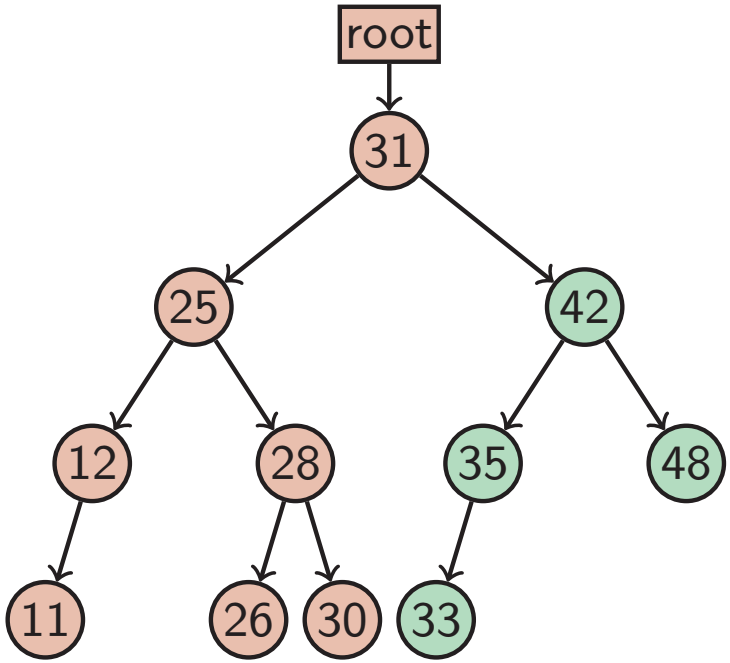
In-order traversal

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8  }
```

```
11 12 25 26 28 30 31 33 35 42 48
```

In-order traversal

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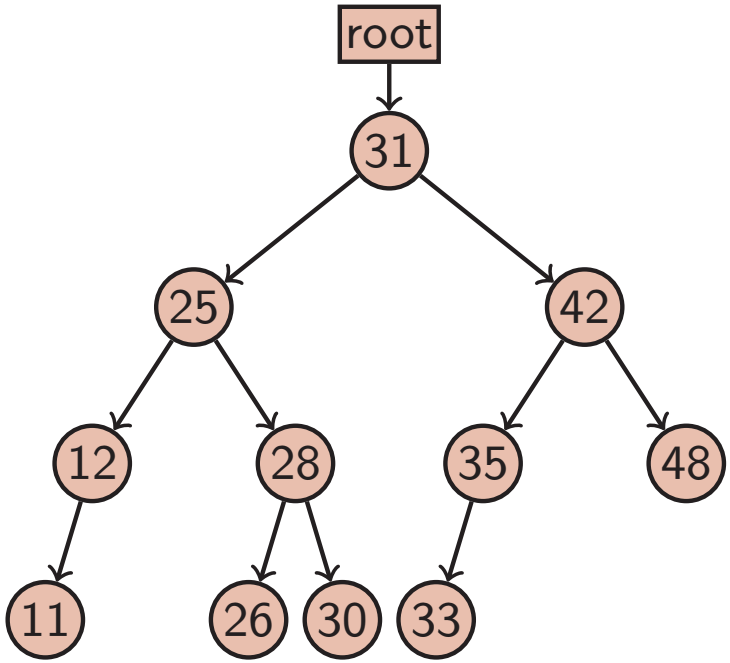


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5    inorder(root->left);
6    printf("%d ", root->data);
7    inorder(root->right);
8  }
```

```
11 12 25 26 28 30 31 33 35 42 48
```

- in-order traversal
  1. left sub-tree
  2. root element
  3. right sub-tree

With this traversal the nodes are visited in increasing order of their values

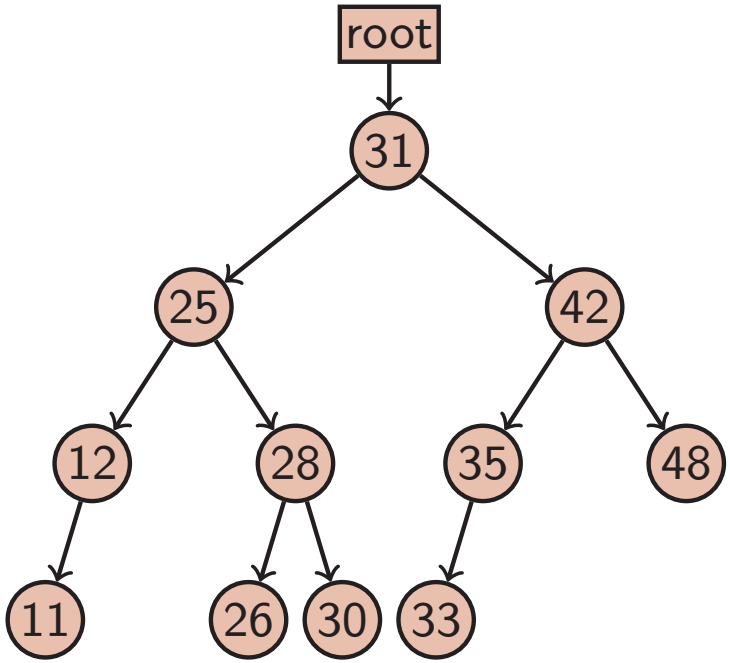
In-order traversal

An other implementation of the in-order traversal:

```
1  void inorder(tree_ptr root)
2  {
3    if (root->left != NULL)
4      inorder(root->left);
5    printf("%d ", root->data);
6    if (root->right != NULL)
7      inorder(root->right);
8  }
```

But in this case the caller has not make sure that `root != NULL` holds

Pre-order traversal




```
1  void preorder(tree_ptr root)
2  {
3    if (root == NULL)
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5    printf("%d ", root->data);
6    preorder(root->left);
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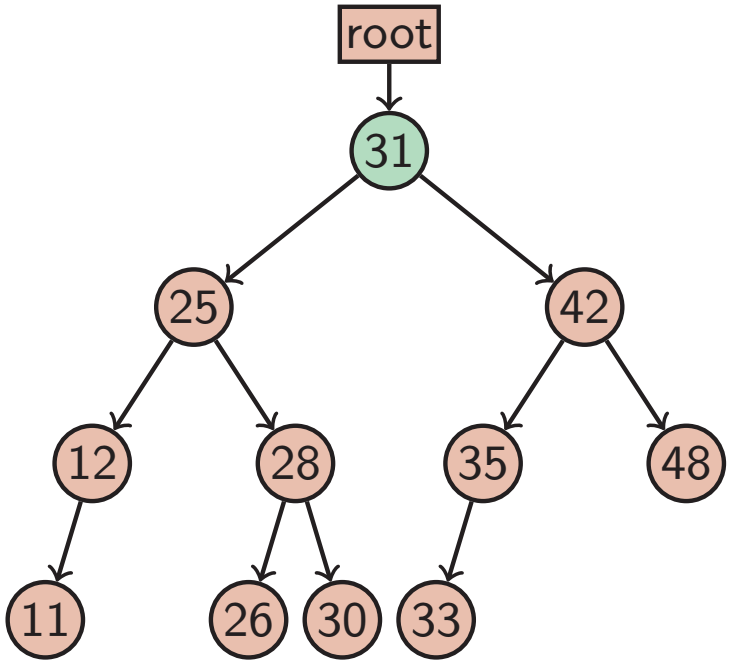
8  }
```

Pre-order traversal

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```
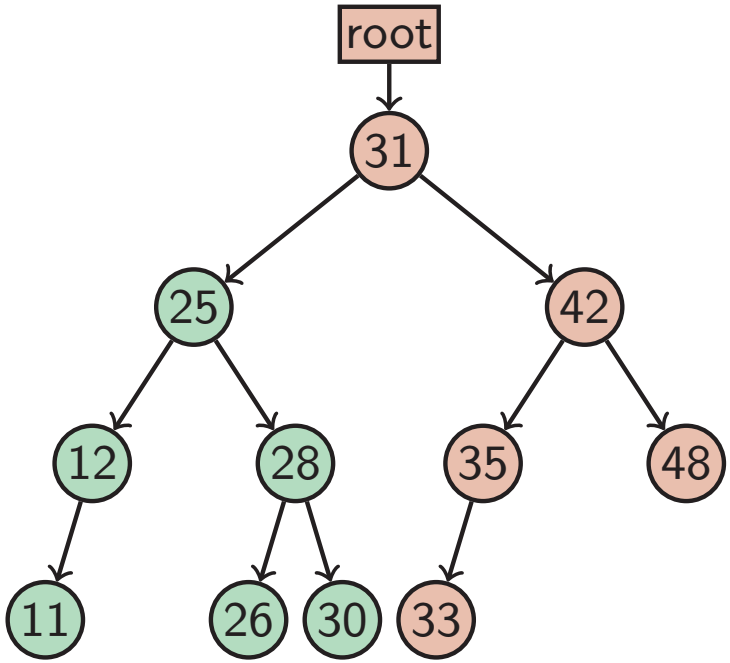

```
31
```

Pre-order traversal

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```
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```

Pre-order traversal

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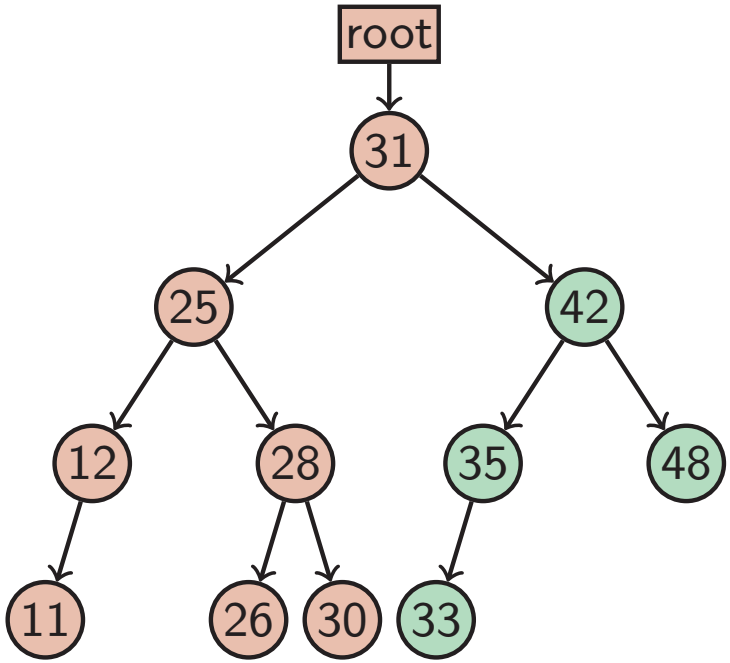


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```
31 25 12 11 28 26 30 42 35 33 48
```

Pre-order traversal

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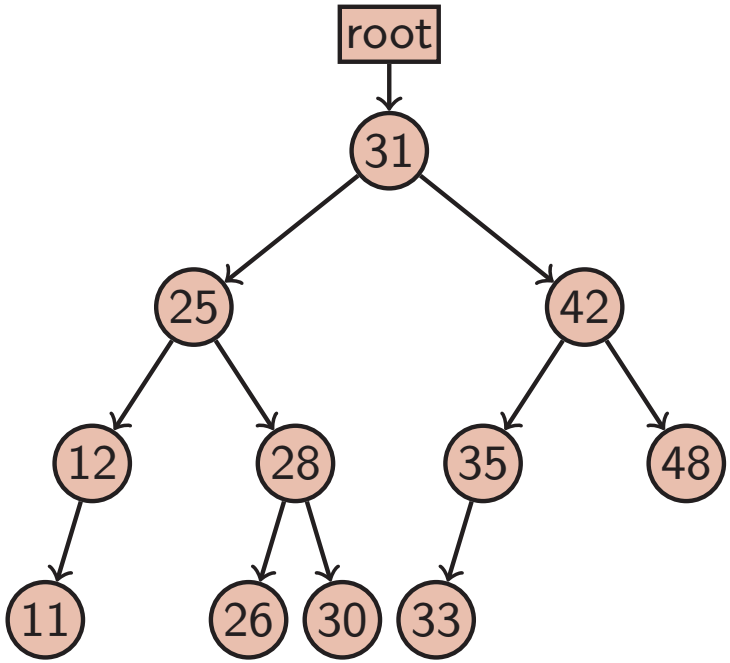


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```

- pre-order traversal
  1. root element
  2. left sub-tree
  3. right sub-tree

Saving the elements of the tree in this order, and building it again, the structure of the tree can be fully reconstructed.

Building a tree

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Inserting a new node to the tree

```
1  tree_ptr insert(tree_ptr root, int data)
2  {
3    if (root == NULL) {
4      root = (tree_ptr)malloc(sizeof(tree_elem));
5      root->data = data;
6    }
7    else if (data < root->data)
8      root->left = insert(root->left, data);
9    else
10     root->right = insert(root->right, data);
11   return root;
12 }
```

link

Building a tree

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

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8      root->left = insert(root->left, data);
9    else
10     root->right = insert(root->right, data);
11   return root;
12 }
```
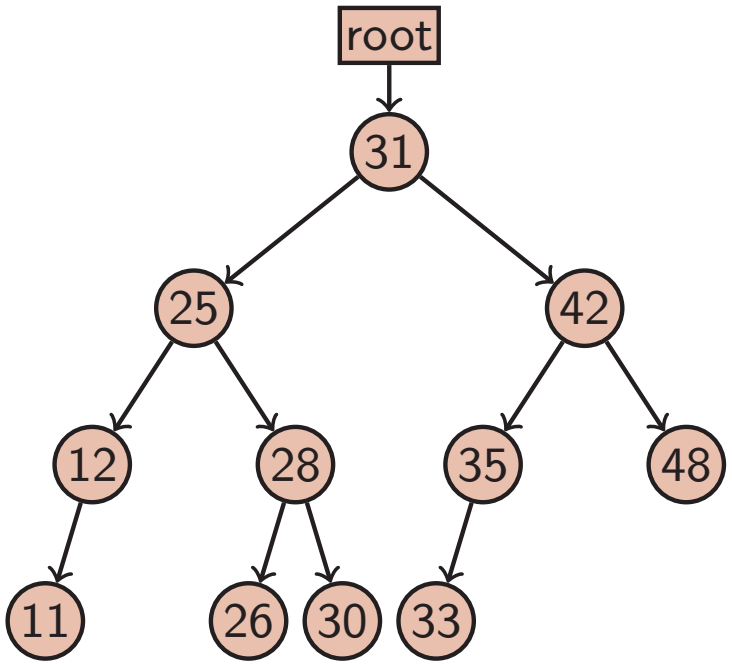

link

Usage of this function:

```
1  tree_ptr root = NULL;
2  root = insert(root, 2);
3  root = insert(root, 8);
4  ...
```

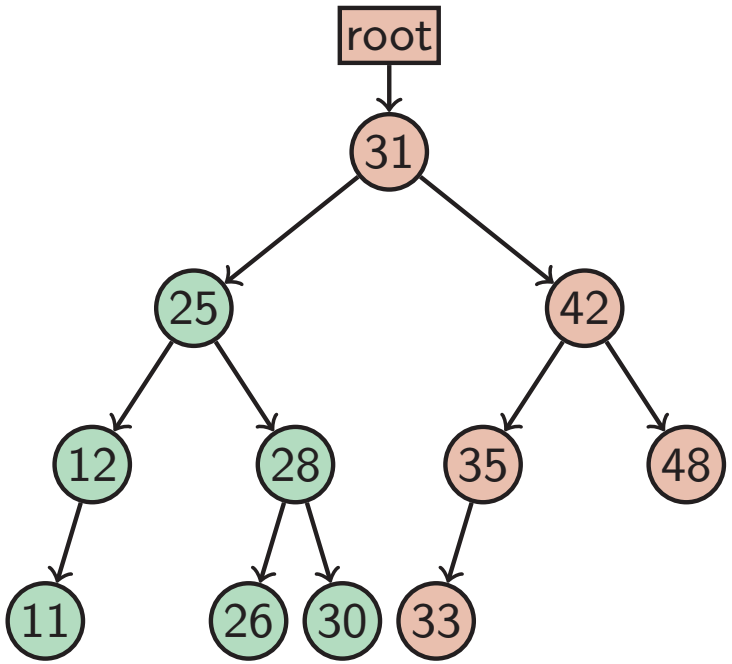
Post-order traversal




```
1  void postorder(tree_ptr root)
2  {
3    if (root == NULL)
4      return;
5    postorder(root->left);
6    postorder(root->right);
7    printf("%d ", root->data);
8  }
```

Post-order traversal

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

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```
11 12 26 30 28 25
```

Post-order traversal

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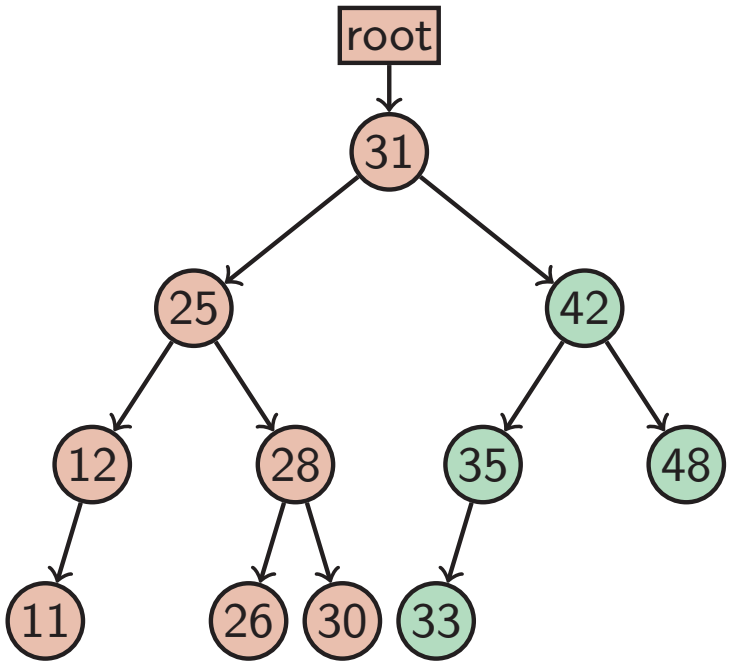


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```

Post-order traversal

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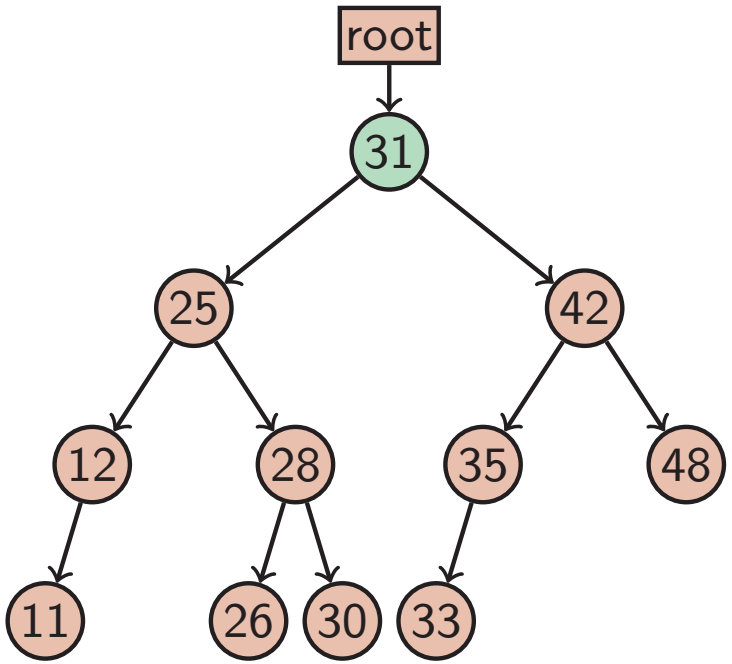


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Post-order traversal

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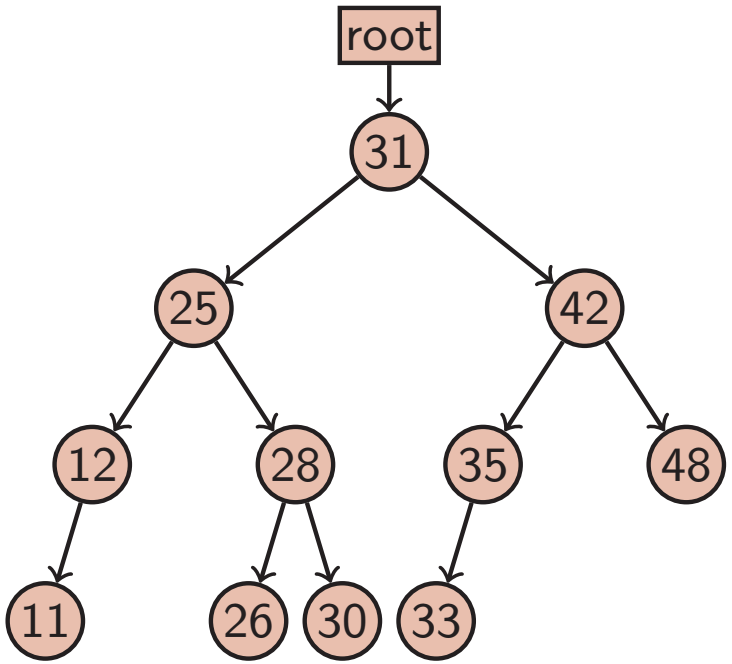


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7    printf("%d ", root->data);
8  }
```

```
11 12 26 30 28 25 33 35 48 42 31
```

- post-order traversal
  1. left sub-tree
  2. right sub-tree
  3. root element

Post-order traversal

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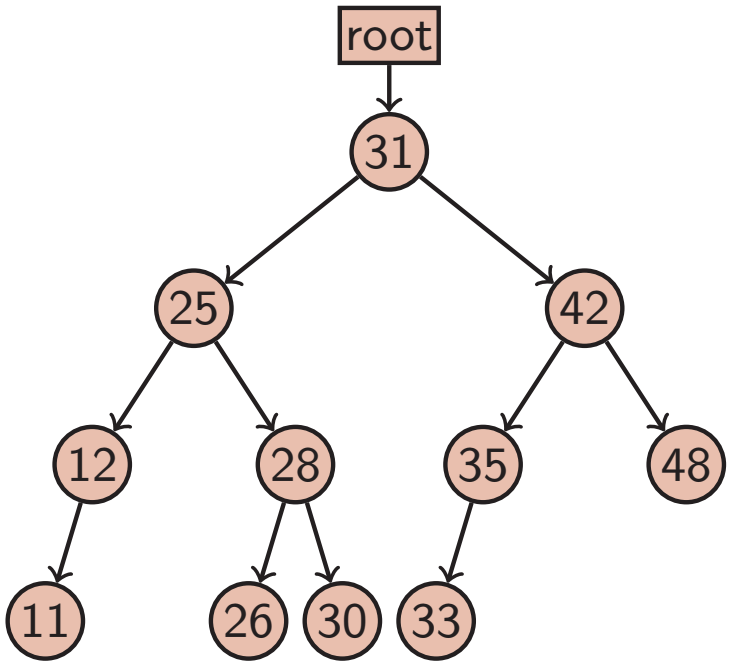


```
1  void postorder(tree_ptr root)
2  {
3    if (root == NULL)
4      return;
5    postorder(root->left);
6    postorder(root->right);
7    printf("%d ", root->data);
8  }
```

```
11 12 26 30 28 25 33 35 48 42 31
```

- post-order traversal
  1. left sub-tree
  2. right sub-tree
  3. root element

In this order the leaves of the tree are visited first → application: releasing/deleting a tree

Deleting a tree by post-order traversal

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```c
void delete(tree_ptr root)
{
  if (root == NULL) /* empty tree: nothing to delete */
    return;
  delete(root->left);    /* post-order traversal */
  delete(root->right);
  free(root);
}
```

link

Deleting a tree by post-order traversal

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void delete(tree_ptr root)
{
  if (root == NULL) /* empty tree: nothing to delete */
    return;
  delete(root->left);    /* post-order traversal */
  delete(root->right);
  free(root);
}
```

link

A program segment (without memory leaks):

```c
tree_ptr root = NULL;
root = insert(root, 2);
root = insert(root, 8);
...
delete(root);
root = NULL;
```

Simple algorithms on binary trees

- Write a recursive function (max. 10 lines), that

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- Write a iterative function (max. 10 lines), that
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  - returns the pointer to the node storing the maximal / minimal value of the tree

Deleting an element from a search tree – naively

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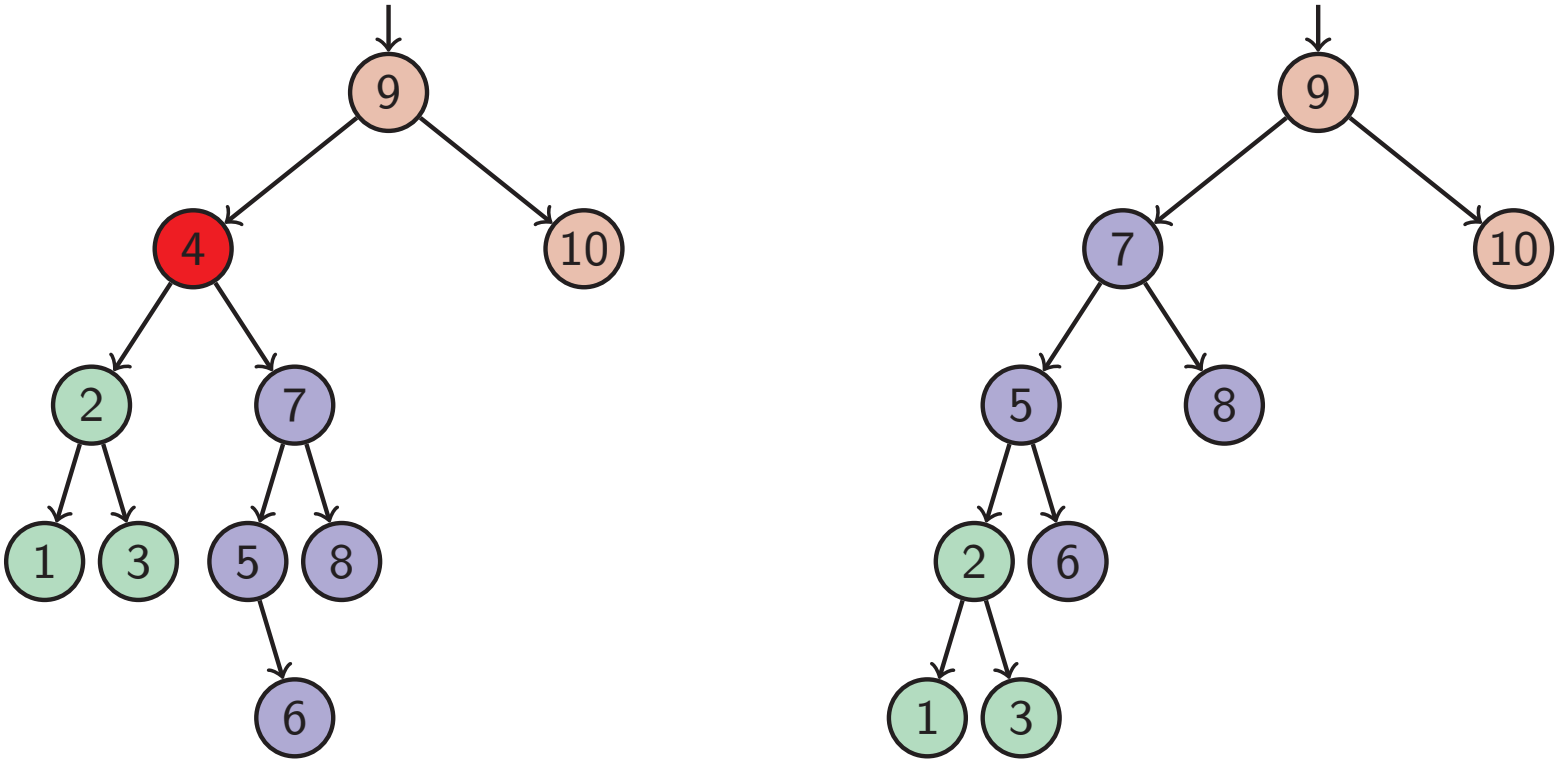

- The right sub-tree is moved to the place of the deleted node
- The left sub-tree is inserted to below the minimal element of the right sub-tree

Deleting an element from a search tree – naively

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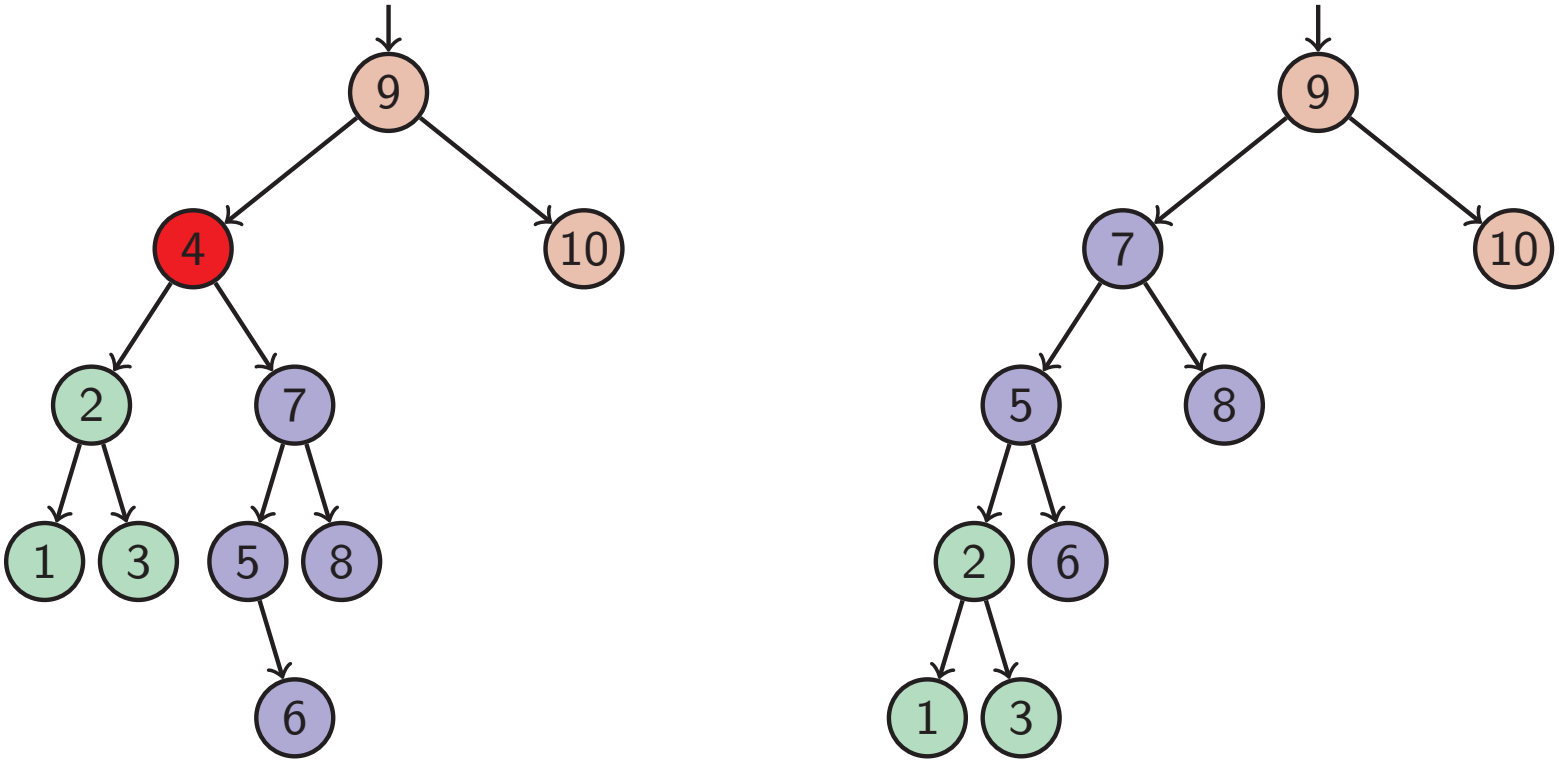

- The right sub-tree is moved to the place of the deleted node
- The left sub-tree is inserted to below the minimal element of the right sub-tree
- The tree is getting imbalanced!

Deleting an element from a search tree – clever

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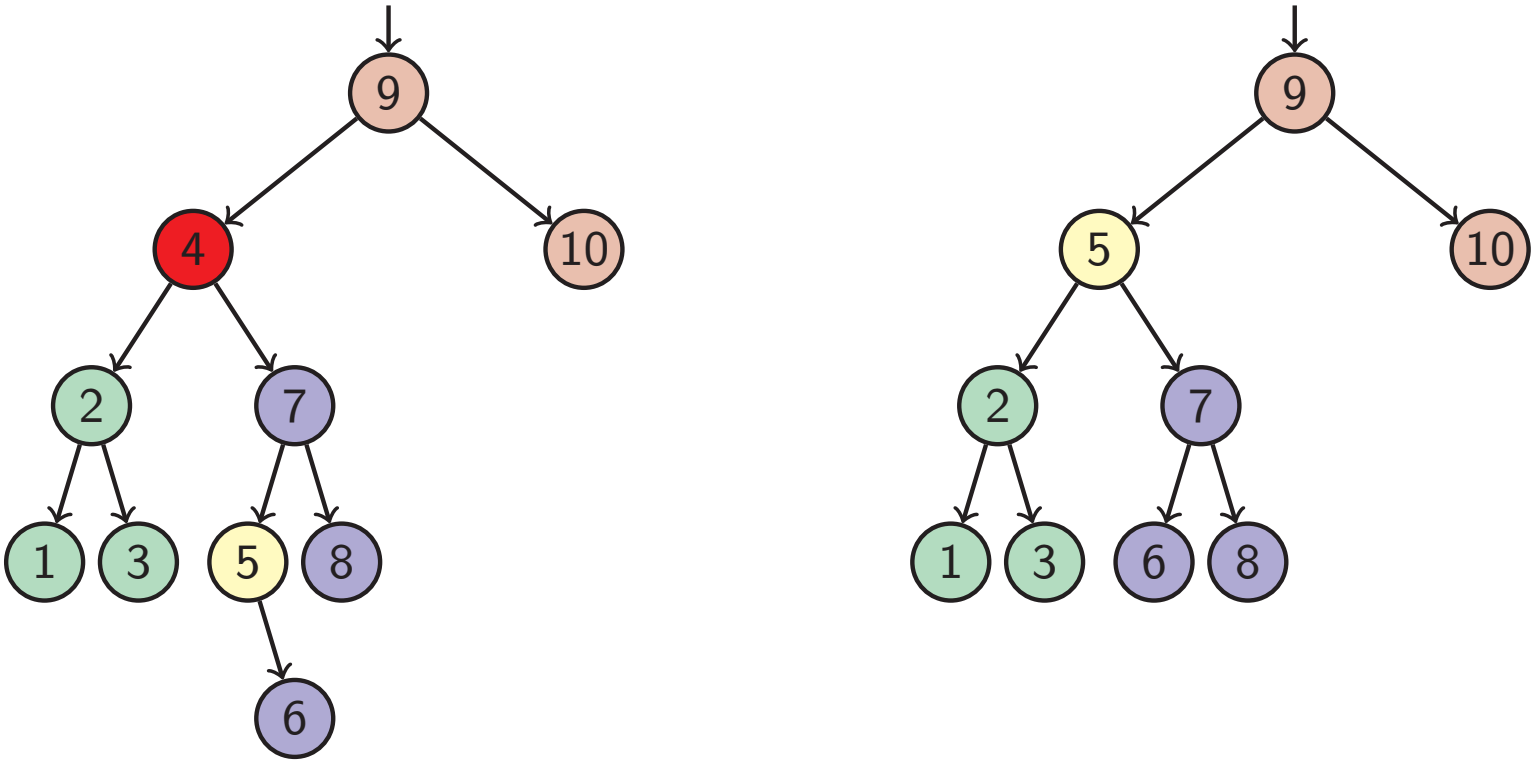

- The minimal element of the right sub-tree is moved to the place of the deleted node
- This element could have only a right sub-tree, it is moved one level up, to its old place

Morse decoding tree

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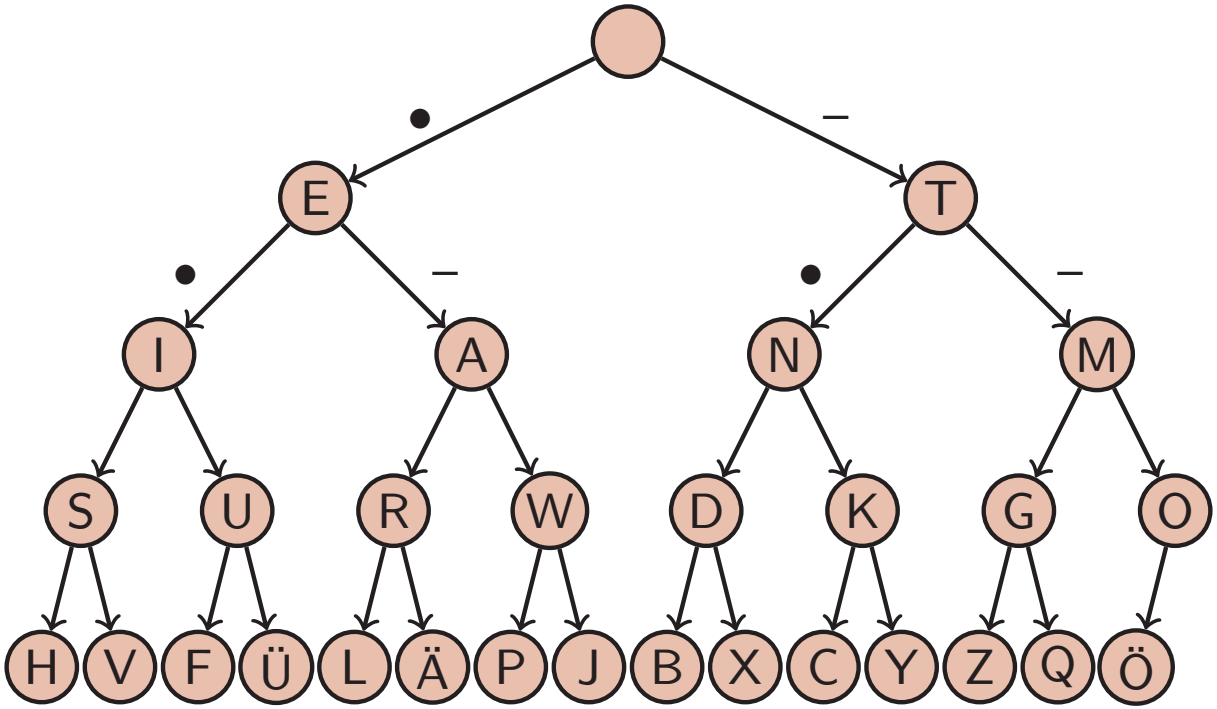

SOSOS: ••• – – – ••• – – – •••

Morse decoding tree

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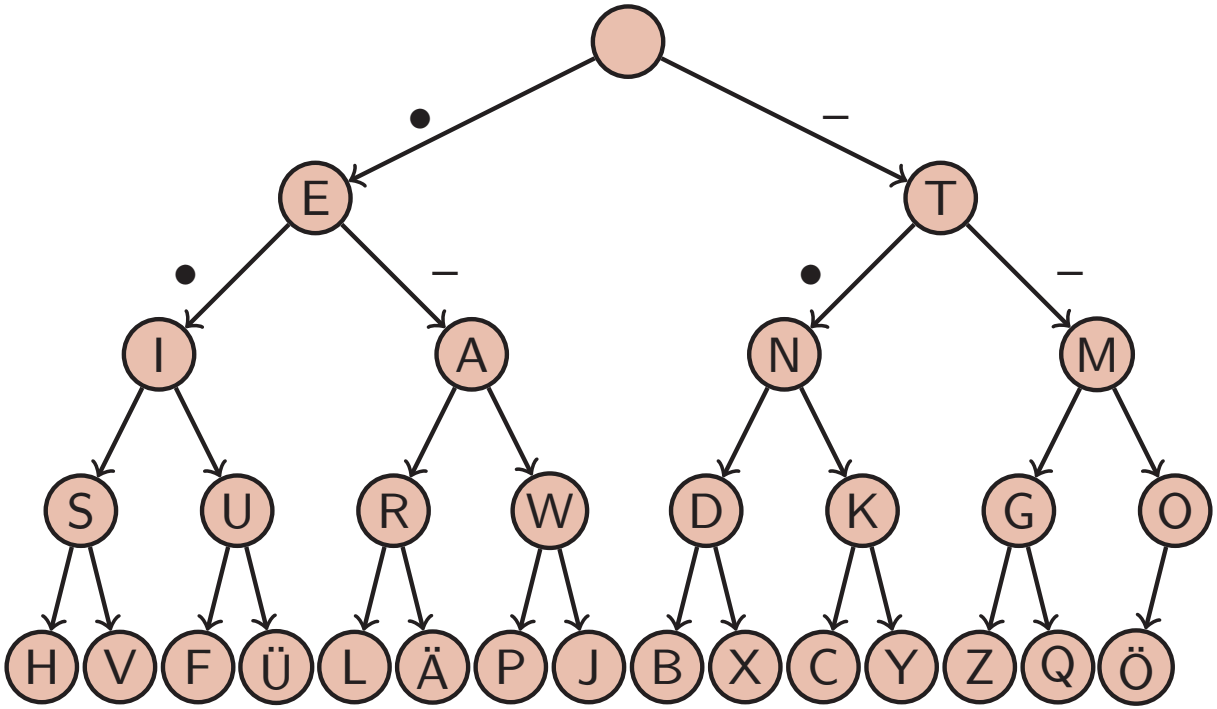

SOSOS: ●●● – – – ●●● – – – ●●●

The response: ●●●● ●– ●●●● ●– ●●●● ●–

Evaluating mathematical expressions

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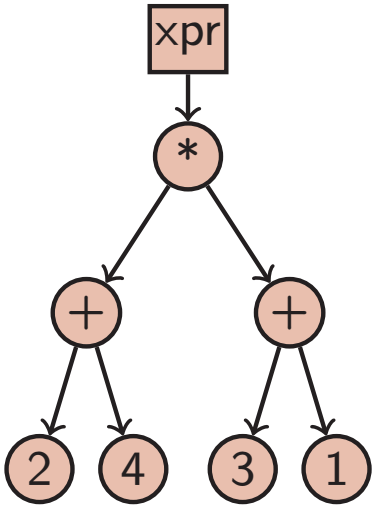

- Storing math expressions in a tree
- Leaves $\rightarrow$ numeric constants
- Branches $\rightarrow$ two-operand operators
- In the example: $(2 + 4) * (3 + 1)$

Evaluating mathematical expressions

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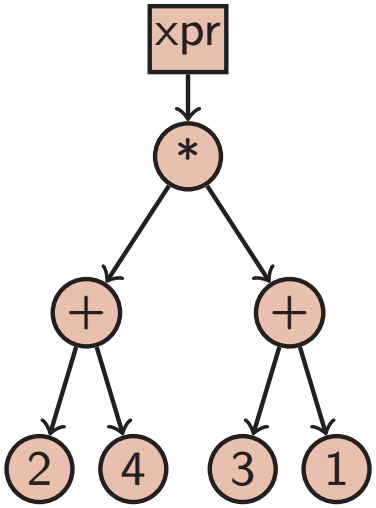

- Storing math expressions in a tree
- Leaves $\rightarrow$ numeric constants
- Branches $\rightarrow$ two-operand operators
- In the example: $(2 + 4) * (3 + 1)$

```
1  int eval(tree_ptr xpr)
2  {
3    char c = xpr->data;
4    if (isdigit(c))      /* stopping condition */
5      return c - '0';
6    if (c == '+')
7      return eval(xpr->left) + eval(xpr->right);
8    if (c == '*')
9      return eval(xpr->left) * eval(xpr->right);
10 }
```

link

Evaluating functions

Let us introduce variable x as a leaf node as well:

```
1  double feval(tree_ptr xpr, double x)
2  {
3    char c = xpr->data;
4    if (isdigit(c))
5      return c - '0';
6    if (c == 'x')
7      return x;
8    if (c == '+')
9      return feval(xpr->left, x) + feval(xpr->right, x);
10   if (c == '*')
11     return feval(xpr->left, x) * feval(xpr->right, x);
12 }
```

link

Evaluating the derivative of a function

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Let us take the derivative of the function! The rules are:

- $c' = 0$
- $x' = 1$
- $(f + g)' = f' + g'$
- $(f \cdot g)' = f' \cdot g + f \cdot g'$

```
1  double deval(tree_ptr xpr, double x)
2  {
3    char c = xpr->data;
4    if (isdigit(c))      /* stopping condition */
5      return 0.0;
6    if (c == 'x')      /* stopping condition */
7      return 1.0;
8    if (c == '+')
9      return deval(xpr->left, x) + deval(xpr->right, x);
10   if (c == '*')
11     return deval(xpr->left, x) * feval(xpr->right, x) +
12       feval(xpr->left, x) * deval(xpr->right, x);
13 }
```

link

Thank you for your attention.