Calculations 1.

Problem 1.

Determine the *Norton* and *Thévenin equivalent* of the voltage divider in Figure 1. using the voltage and current directions given.

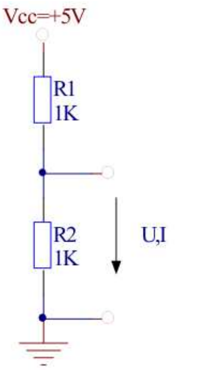

Figure 1. *Voltage divider circuit*

Solution

Let us determine the output voltage when the load is an open circuit by using the well-known equation of the voltage divider:

$$U_G = V_{CC} \cdot \frac{R_2}{R_1 + R_2} = 2.5\,\mathrm{V}$$

Next we shall calculate the output current when the load is a short circuit. If the output ports are shorted, so is resistor R_2. Thus the supply voltage is connected to the ground through R_1, the current of which can be determined using Ohm's law:

$$I_G = \frac{V_{CC}}{R_1} = 5\,\mathrm{mA}$$

The value of the *internal impedance* is:

$$R_G = \frac{U_G}{I_G} = 500\,\Omega$$

The equivalent circuits can be seen in Figure 2.

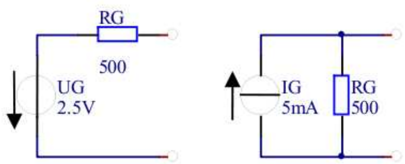

Figure 2. *The Thévenin and Norton equivalents*

Problem 2.

Find appropriate values for the resistor R and the capacitor C in order to have a 100 ms long logic low value on the $\overline{\text{RESET}}$ input of the microcontroller when the supply voltage of 3.3 V is switched on. The threshold voltage of the $\overline{\text{RESET}}$ input is 60% of the supply voltage. The threshold voltage is the value below which the logic circuit determines the input as a logic low and above it as a logic high.

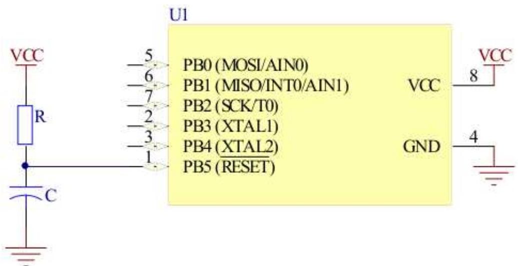

Figure 3. *Delay circuit on a reset input*

Solution

The capacitor's voltage as a function of time is:

$$V_C(t) = V_{CC} \cdot \left(1 - \mathrm{e}^{-\frac{t}{R \cdot C}}\right)$$

Let us determine the time it takes for the voltage to rise to the 60% of the supply voltage:

$$0.6 \cdot V_{CC} = V_{CC} \cdot \left(1 - \mathrm{e}^{-\frac{t}{R \cdot C}}\right)$$

$$0.4 = \mathrm{e}^{-\frac{t}{R \cdot C}}$$

Let us calculate the logarithm of both sides:

$$\ln 0.4 = -\frac{t}{R \cdot C}$$

Using the following logarithmic identity: $a \cdot \log(b) = \log\left(b^a\right)$

$$t = R \cdot C \cdot \ln(2.5)$$

By using the approximation: $\mathrm{e} \approx 2.5$

$$R \cdot C = 100\,\mathrm{ms}$$

A possible solution for the problem is: $R = 47\,\mathrm{k}\Omega$, $C = 2.2\,\mu\mathrm{F}$.

Problem 3.

Determine the current of the diode in Figure 4. using

(a) the $V_D = 0.7\,\text{V}$ approximation.

(b) the more accurate model that was shown in class.

The model parameters of the 1N914 diode are the following: $I_0 = 2.52\,\text{nA}$, $R_S = 0.568\,\Omega$, $m = 1.752$. Determine the error caused by the approximation.

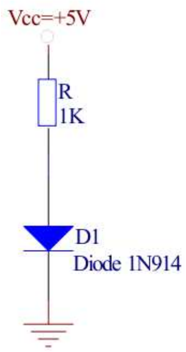

Figure 4. *Simple diode circuit*

Solution

Using the $V_D = 0.7\,\text{V}$ approximation, the current of the diode is

$$I_D = \frac{V_{CC} - V_D}{R} = 4.3\,\text{mA}$$

For a more accurate calculation, let us use a more precise diode characteristics:

$$V_D = m \cdot V_T \cdot \ln\left(\frac{I}{I_S} + 1\right) + I \cdot R_S$$

Using Kirchhof's voltage law (KVL):

$$V_{CC} = I \cdot R + V_D = m \cdot V_T \cdot \ln\left(\frac{I}{I_S} + 1\right) + I \cdot (R + R_S)$$

This equation has to be solved. By solving it with Excel we get the following results: $I = 4.34\,\text{mA}$, $V_D = 0.656\,\text{V}$. We were able to determine the current with a precision of 1% and the voltage with 7%.

Problem 4.

Determine the transfer characteristics of the circuits in Figure 5. All the resistors have a resistance of $1\,\mathrm{k}\Omega$ and the $V_D = 0.7\,\mathrm{V}$ approximation can be used for the diodes.

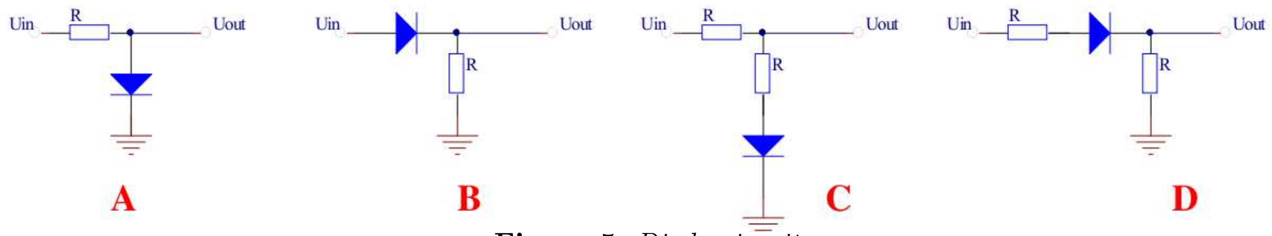

Figure 5. *Diode circuits*

Solution

The point where the diode starts to conduct has to be found in each case:

(a) If the input voltage is less than $0.7\,\mathrm{V}$ the diode is closed and the output voltage is equal to the input. If the input voltage is above $0.7\,\mathrm{V}$ then the diode conducts and the output voltage equals to the diode's voltage ($0.7\,\mathrm{V}$). The rest of the input voltage is dropped on the resistor.

(b) If the input voltage is less than $0.7\,\mathrm{V}$ the diode is closed and the output voltage is $0\,\mathrm{V}$ (as the resistor pulls the output voltage down to the ground). When the input voltage rises above $0.7\,\mathrm{V}$ the diode starts to conduct and the output voltage is: $V_{out} = V_{in} - 0.7$

(c) If the input voltage is less than $0.7\,\mathrm{V}$ the diode is closed and the output voltage equals to the input voltage. If the input votage is above $0.7\,\mathrm{V}$ the diode is open and $V_{in} - 0.7\,V$ is dropped on the two resistors of equal resistance. The output voltage equals to the sum of the voltage on the lower resistor and the diode:

$$V_{out} = \frac{(V_{in} - 0.7)}{2} + 0.7$$

(d) If the input voltage is lower than $0.7\,\mathrm{V}$ than the diode is closed and the output voltage is $0\,\mathrm{V}$. If the input voltage is higher than that then the diode conducts and $V_{in} - 0.7\,V$ voltage is dropped on the two resistors thus:

$$V_{out} = \frac{(V_{in} - 0.7)}{2}$$

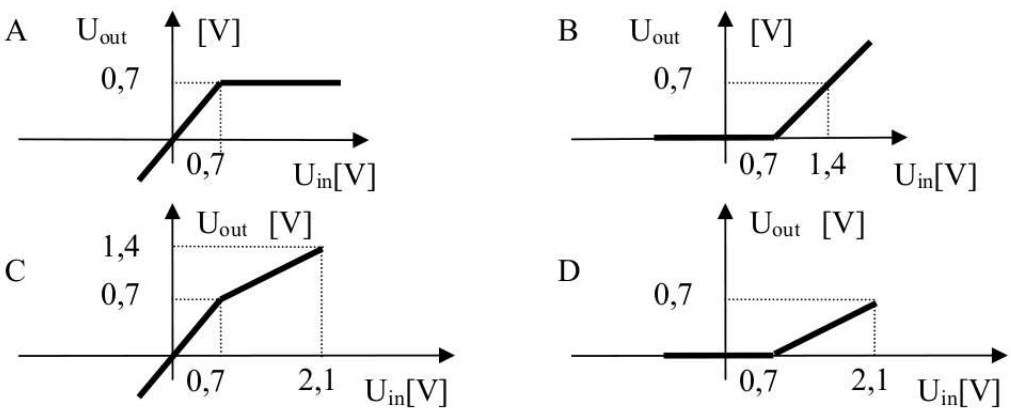

Figure 6. *The transfer characteristics of the circuits in Figure 5.*

Problem 5.

In Figure 7. a voltage reference generator circuit using a Zener diode can be seen. The breakdown voltage of the diode is 10 V, its differential resistance is 10 Ω. The resistor R has a resistance of 1 kΩ and the unstable supply voltage is 15 V. The output is not loaded.

(a) Determine the output voltage.

(b) Calculate the output voltage's ripple if that of the input voltage is 1 V.

(c) Calculate the *Power Supply Rejection Ratio* (PSRR) in dB.

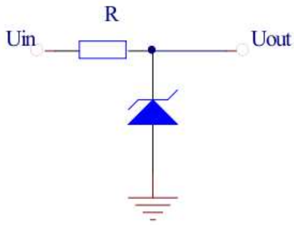

Figure 7. *Reference voltage generator circuit*

Solution

(a) Let us use the equivalent circuit of the Zener diode for the breakdown range. The current based

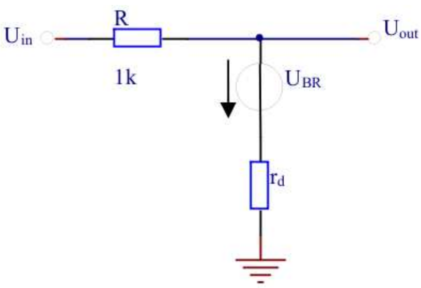

Figure 8. *Reference voltage generator with the equivalent circuit of the Zener diode*

on Figure 8:

$$I = \frac{V_{in} - V_{br}}{R + r_d}$$

The output voltage is equal to the voltage dropped on the Zener diode:

$$V_{out} = V_{br} + r_d \cdot I = V_{br} + \frac{r_d}{R + r_d} \cdot (V_{in} - V_{br}) = \frac{R}{R + r_d} \cdot V_{br} + \frac{r_d}{R + r_d} \cdot V_{in} = 10.05\,\mathrm{V}$$

By deriving the equation above and applying the approximation: $r_d \ll R$

$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{r_d}{R + r_d} \approx \frac{r_d}{R}$$

thus

$$\Delta V_{out} \approx \frac{r_d}{R} \cdot \Delta V_{in} = 10\,\mathrm{mV}$$

(b) The PSRR is the reciprocal of the above value:

$$PSSR = 20 \cdot \lg \frac{R}{r_d} \approx 40\,\mathrm{dB}$$

Problem 6.

Using the $V_D = 0.7\,\mathrm{V}$ approximation determine the transfer characteristics of the circuit below.

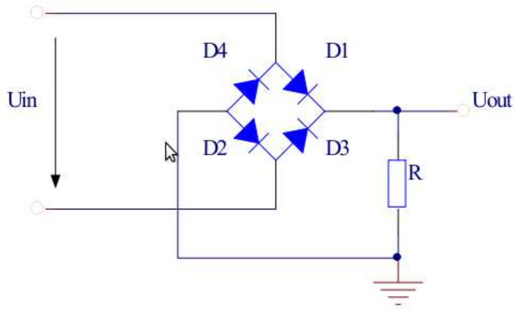

Figure 9. *A full-wave rectifier*

Solution

The diodes D_1–D_4 form a full-wave rectifier. For positive input voltages, the D_1 and D_2 diodes are open and the D_3 and D_4 are closed. For negative input voltages it's the other way around.

Current can only flow through the resistor R if the absolute value of the input voltage is bigger than the threshold voltage of two diodes. The structure of the diodes is such that whatever the sign of the input voltage, the current of the resistor will always flow in the same direction thus the voltage drop on the resistor will always have the same sign. This is how the rectifying is accomplished.

Hence the transfer characteristics is the following:

$$V_{out}\left(V_{in}\right) = \begin{cases} 0 & if\,|V_{in}| < 2 \cdot V_D \approx 1.4\,\mathrm{V} \\ |V_{in}| - 2 \cdot V_D & if\,|V_{in}| \geq 2 \cdot V_D \end{cases}$$

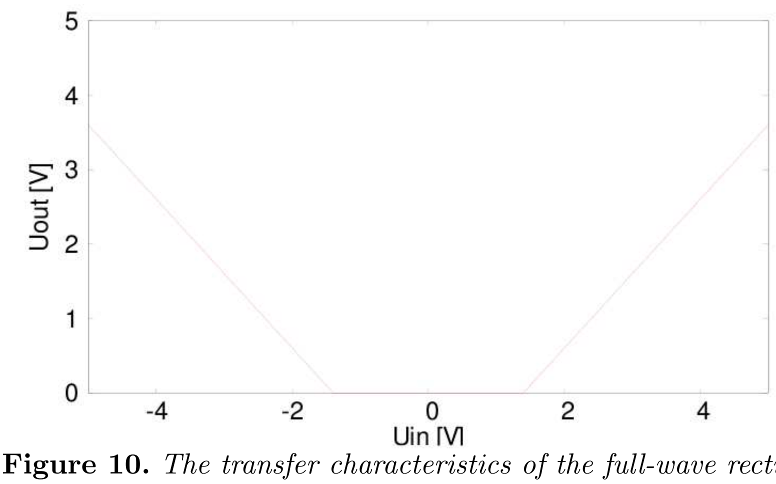

Figure 10. *The transfer characteristics of the full-wave rectifier*

The output current will be the following:

$$I_{out}\left(V_{in}\right) = \begin{cases} 0 & if\,|V_{in}| < 2 \cdot V_D \approx 1.4\,\mathrm{V} \\ \frac{|V_{in}| - 2 \cdot V_D}{R} & if\,|V_{in}| \geq 2 \cdot V_D \end{cases}$$